G.C.E. A/L Examination November - 2015

Conducted by Field Work Centre, Thondaimanaru

In Collaboration with

Zonal Department of Education Jaffna.

FWC

Grade :- 12 (2017)	Combined Mathematics	Time :- Three hours

Instructions

- This question paper consists of two parts; Part A (questions 1 -10) and part B (questions 11- 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the question paper. If additional space is needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined Maths		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Combined Maths	
Final Marks	

Part – A

(1) Let $f(x) = 2x^2 - 4kx + 2k^2 + k - 1$.

If the minimum value of $f(x)$ is 2, find the value of k.

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(2) Find the set of real values of x satisfying the inequality $\frac{3x-1}{x} < 1$.

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(3) Express $\frac{3x}{(x-1)(x+2)}$ as partial fractions.

(4) Of $\frac{4a-b}{2a-b} = 7$ › find the value of $\frac{3a+b}{a+b}$.

(5) Let $sin \propto = \frac{1}{\sqrt{10}}$ and $cos\,\beta = -\frac{1}{\sqrt{5}}$ where $\frac{\pi}{2} < \propto < \pi$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of $sin\,(\propto + \beta)$.

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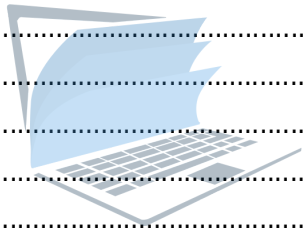

(6) The resultant of two forces acting at an angle 45^0 is $\sqrt{10}\ N$. The resultant of these forces acting at an angle 90^0 is $\sqrt{6}\ N$. Find these forces.

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(7) $ABCDEF$ is a regular hexagon. Forces of magnitudes 2, $4\sqrt{3}$, 10 and 6 N act along $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{DA}$ and $\overrightarrow{AF}$ respectively. Show that the system of forces is in equilibrium.

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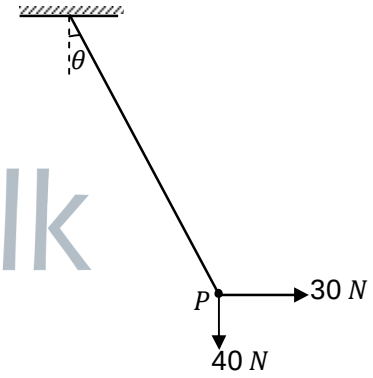
(8) As shown in figure, a particle P of weight 40 N is suspended by a string and it is pulled aside by a horizontal force 30 N. If the inclination of the string to the vertical is θ and the tension of the string is T, using Lami's theorem, show that

(i) $\tan\theta = \frac{3}{4}$ *and*

(ii) $T = 50\ N$.



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(9) Let $\boldsymbol{i}$ and $\boldsymbol{i}-\boldsymbol{j}$ be the position vectors of two points A and B respectively with respect to origin O Also, let C be the point such that AC is parallel to OB. Show that $\overrightarrow{oc} = (1+\lambda)\,\boldsymbol{i} - \lambda\,\boldsymbol{j}$, where λ is a real number.

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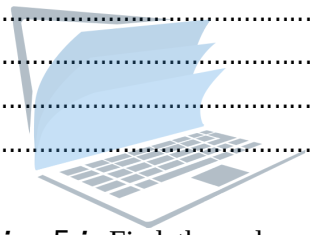

(10) Let $\boldsymbol{a} = 2\,\boldsymbol{i} + \lambda\,\boldsymbol{j}$ and $\boldsymbol{b} = 3\,\boldsymbol{i} - 5\,\boldsymbol{j}$. Find the value of λ such that b and $2\boldsymbol{a} + \boldsymbol{b}$ are perpendicular vectors to each other.

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Part - B

(11) a) $\propto$, and β are the roots of the quadratic equation $\lambda(x^2 - x) + 2(x + 1) = 0$, where $\lambda \neq o$. Find $\frac{\propto}{\beta} + \frac{\beta}{\propto}$ in terms of λ .

If λ_1 and λ_2 are two values of λ such that $\frac{\propto}{\beta} + \frac{\beta}{\propto} = -3\lambda$ without finding λ_1, λ_2 separately , show that $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_2} = -1$.

b) If the equation $a^2x^2 + 6abx + ac + 8b^2 = 0 \quad (a \neq 0)$ has equal roots, show that the roots of the equation $ac\,(x + 1)^2 = 4b^2x$ are equal.

c) State and prove the Remainder theorem. When $2x^4 + x^3 - x^2 + ax + b$ is divided by $(x^2 - 1)$ the remainder is $2x + 3$. Find the values of a and b

(12) a) Let $b = a + d$ and $c = b + d$ for $a, b, c, d \in \mathcal{R}^+$.
Show that $\log(ac + d^2) = 2\log b$

b) If $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5}$ show that

i) $xy = z$ and

ii) $x^4 = yz$.

c) Solve the following equations.

i) $2^{2x} - 3.2^{x+2} + 32 = 0$

ii) $\sqrt{x + 3} - \sqrt{2x - 1} = 1$

(13) a) Show that

(i) $2\,[sin^6x + cos^6x] - 3[sin^4x + cos^4x] + 1 = 0$ and

(ii) The expression $cos^2\theta + cos^2(\propto + \theta) - 2\,cos \propto \cos\theta\ cos(\propto + \theta)$ does not depend on θ

b) Prove that $\frac{Sin\,\theta}{Cos\,3\,\theta} = \frac{1}{2}(\tan 3\theta - tan\,\theta)$.

Hence, deduce that

$\frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}[\tan 27\,\theta - \tan\theta]$.

c) If $A + B + C = 180^0\lambda$ Show that

$\cos A + \cos B + \cos C = 1 + 4\,sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

(14) a) Considering the expansion of $\tan(A + B)$,

Show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

Show that $\tan\theta \tan(60 - \theta) \tan(60 + \theta) = \tan 3\theta$.

Hence, deduce that $\tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0 = 3$.

b) Find the general solutions of the following equations.

(i) $\sin 2\theta - \cos 2\theta - \sin\theta + \cos\theta = 0$

(ii) $\tan\theta + \sec\theta = \sqrt{3}$

c) If $t = \tan\frac{\theta}{2}$, show that $\cos\theta = \frac{1 - t^2}{1 + t^2}$ and $\sin\theta = \frac{2t}{1 + t^2}$.

$\cos\theta = \cos\propto \cos\beta$ show that

$\tan\left(\frac{\theta + \propto}{2}\right) \tan\left(\frac{\theta - \propto}{2}\right) = \tan^2\frac{\beta}{2}$.

(15) a) Two forces P and Q act at a point. If the resultant of them divides the angle between P, and Q in the ratio $1 : 2$, show that the resultant is $\frac{P^2 - Q^2}{Q}$.

Show that the angle between P and Q is $3\cos^{-1}\left(\frac{P}{2Q}\right)$.

b) Two forces $3P$ and $2P$ act at a point. When the first is doubled and $20\ N$ is added to the second force the direction of resultant is unaltered. Find the value of P .

(16) a) $ABCDEF$ is a regular hexagon. Forces of magnitudes $2, 4\sqrt{3}, 8, 2\sqrt{3}$ and $4\ N$ act along $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE}$, and $\overrightarrow{AF}$ respectively.

i) Find the magnitude of the resultant.

ii) Find the angle made by the resultant with AD .

b) A particle of weight $50\ kg$ is attached to the ends of the strings of lengths $8m$ and $6m$ and other ends attached to two points which are at the same horizontal level at distance $10\ m$ apart, and hangs in equilibrium. Find the tensions in the strings.

(17) a) In triangle ABC, D, E and F are mid – points of BC, CA and AB respectively. Prove the following.

(i) $\overrightarrow{FE} = \frac{1}{2}\overrightarrow{BC}$

(ii) $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$

b) (i) Let $\boldsymbol{a} = 3\boldsymbol{i} - 4\boldsymbol{j}$ and $\boldsymbol{b} = \lambda\boldsymbol{i} + 6\boldsymbol{j}$.

If $a \perp b$, find λ .

(ii) If $|\boldsymbol{a}| = |\boldsymbol{b}| = |\boldsymbol{a} - \boldsymbol{b}|$, find the angle. Between a, and b by scalar product.