வடமாகாணக் கல்வித் திணைக்களத்துடன் இணைந்து தொண்டைமானாறு
வெளிக்கள நிலையம் நடாத்தும் தவணைப் பரீட்சை, மார்ச் - 2020

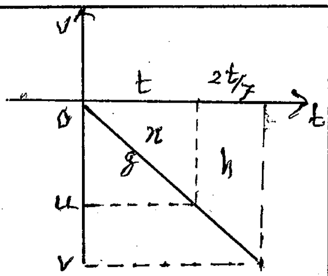
Conducted by Field Work Centre, Thondaimanaru

In Collaboration with Provincial Department of Education Northern Province

Term Examination, March - 2020

Grade - 13 (2020) | **Combined Maths II** | **Marking Scheme**

(1)



$t = \frac{u}{g}$

$t + \frac{2t}{7} = \frac{v}{g}$

$h = \frac{1}{2}(u+v) \cdot \frac{2t}{7}$ (5)

$= \frac{t}{7}\left[gt + \frac{9gt}{7}\right]$

$= \frac{16gt^2}{49}$

$t = \frac{7}{4}\sqrt{\frac{h}{g}}$ (5)

$AC = h + x$ (5)

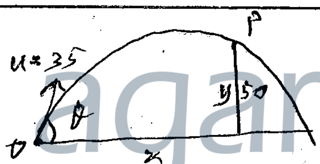
$= h + \frac{1}{2}gt^2$

$= h + \frac{1}{2}g\frac{49}{16}\cdot\frac{h}{g}$

$= \frac{81h}{32}$ (5)

$\boxed{25}$

(2)



$\tan\theta = 2$

$O \to P \quad s = ut + \frac{1}{2}at^2$

$\rightarrow \quad x = u\cos\theta \cdot t$ ——— (1)

$\uparrow \quad y = u\sin\theta t - \frac{1}{2}gt^2$ ——— (2)

$(1)\&(2) \Rightarrow y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}(\sec^2\theta)$ ——— (3) (5)

$50 = x \cdot 2 - \frac{9.8\, x^2 \cdot 5}{2 \cdot 35^2}$ (5)

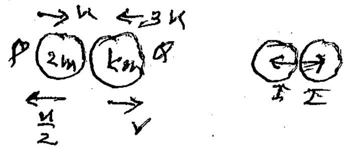
$x^2 - 100x + 2500 = 0$ (5)

$(x-50)^2 = 0$

$x = 50$ (5)

$(1) \Rightarrow t = \frac{10\sqrt{5}}{7}$ (5)

(3)



for $P \rightarrow \underline{I} = \Delta m\underline{v}$

$-I = 2m\left(-\frac{u}{2}\right) - 2mu$

$I = 3mu$

for $Q \rightarrow \underline{I} = kmv - km(-3u)$

$3mu = kmv + 3kmu$ (5)

$v = \frac{3u}{k}(1-k)$ (5)

$v > 0 \Rightarrow \frac{1}{k}(1-k) > 0 \qquad k > 0$

$k < 1 \qquad + \; k > 0$

$\Rightarrow \quad 0 < k < 1.$ (5)

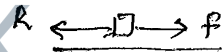
N.R.L $\quad v + \frac{u}{2} = e(u + 3u)$ (5)

$k = \frac{1}{2} \Rightarrow v = 3u$

$\therefore e = \frac{7}{8}$ (5)

$\boxed{25}$

(4)



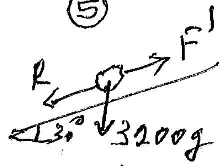
$36 \times 1000 = F \times 20$ (5)

$F = 1800\ N$

$\rightarrow F - R = 3200\,a$ (5)

$1800 - R = 3200 \times 0.2$

$R = 1160\ N$ (5)



$\nearrow \quad F' - R - 3200g\sin 30^\circ = 0$ (5)

$F' = 17160\ N$

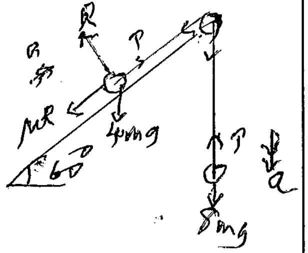
$P = 17160 \times 30$

$= 514800\ W$ (5)

$= 514.8\ kW$

$\boxed{25}$

(5)



$\nearrow R = 4mg\cos 60 = 2mg$ (5)

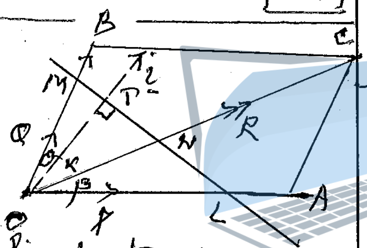
$\nearrow T - \mu R - 4mg\sin 60 = 4ma$ (5)

$Q \downarrow 8mg - T = 8ma$ (5)

$a = \frac{g}{24}(15 - 4\sqrt{3})$ (5)

$T = \frac{g}{3}(9 + 4\sqrt{3})$ (5) $\boxed{25}$

(6)



(5)

OT is a perpendicular to LMN

let $\underline{i}$ is a unit vector in the direction $\overrightarrow{OT}$

$\underline{P} + \underline{Q} = \underline{R}$

$\underline{P}\cdot\underline{i} + \underline{Q}\cdot\underline{i} = \underline{R}\cdot\underline{i}$

$|\underline{P}|.1.\cos(\alpha+\beta) + |\underline{Q}|.1.\cos\theta = |\underline{R}|.1\cos\alpha$ (10)

$P.\frac{OT}{OL} + Q.\frac{OT}{OM} = R.\frac{OT}{ON}$ (5)

$\Rightarrow \frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$ (5) $\boxed{25}$

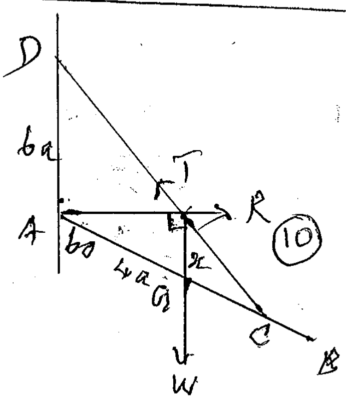
(7)

$x = 2a$

$\frac{AC}{CG} = \frac{AD}{x} = \frac{6a}{2a}$ (10)

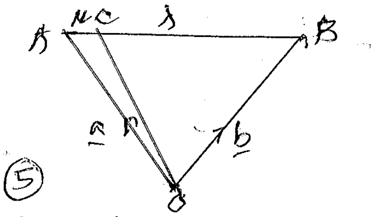
$\therefore CG = 2a$

$\therefore BC = 2a$ (5)



(10) $\boxed{25}$

(8)



$\frac{AC}{CB} = \frac{\mu}{\lambda}$

$\lambda\overrightarrow{AC} = \mu\overrightarrow{CB}$ (5)

$\lambda(\underline{c} - \underline{a}) = \mu(\underline{b} - \underline{c})$ (5)

$(\lambda+\mu)\underline{c} = \lambda\underline{a} + \mu\underline{b}$

$\underline{c} = \frac{\lambda\underline{a} + \mu\underline{b}}{\lambda+\mu}$ (5)

$\alpha\underline{a} + \beta\underline{b} + \gamma\underline{c} = \underline{0}$ & $\alpha+\beta+\gamma = 0$

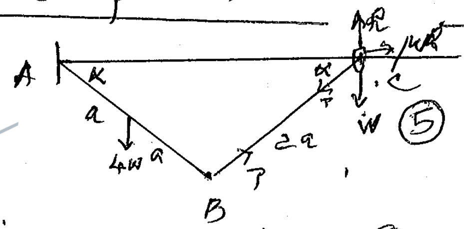
$\alpha\underline{a} + \beta\underline{b} = -\gamma\underline{c}$ $\qquad \alpha+\beta = -\gamma$

$\therefore \frac{\alpha\underline{a} + \beta\underline{b}}{\alpha+\beta} = \frac{-\gamma\underline{c}}{-\gamma}$ (5)

$\Rightarrow \frac{\alpha\underline{a} + \beta\underline{b}}{\alpha+\beta} = \underline{c}$ (5) $\boxed{25}$

from the 1st part A, B, C are Collinear.

(9)



for Ring $\leftarrow T\cos\alpha - \mu R = 0$ (5)

$\uparrow R - W + T\sin\alpha = 0$ (5)

for Rod & Ring A↻

$4a\cos\alpha(R - W) - 4wa\cos\alpha = 0$ (5)

$R = 2W$

$\frac{(2)}{(1)} \Rightarrow \tan\alpha = \frac{R - W}{\mu R} = \frac{1}{2\mu}$ (5)

$\tan\alpha = \frac{1}{2\mu}$ $\boxed{25}$

(10) $P(A\cap B') = \frac{8}{25}$, $P(A'\cap B) = \frac{11}{100}$

$P(A\cup B) = \frac{13}{20}$

I $P(A\cup B) = P(A\cap B') + P(A'\cap B) + P(A\cap B)$ (5)

$\frac{13}{20} = \frac{8}{25} + \frac{11}{100} + P(A\cap B)$

$P(A\cap B) = \frac{22}{100}$ (5)

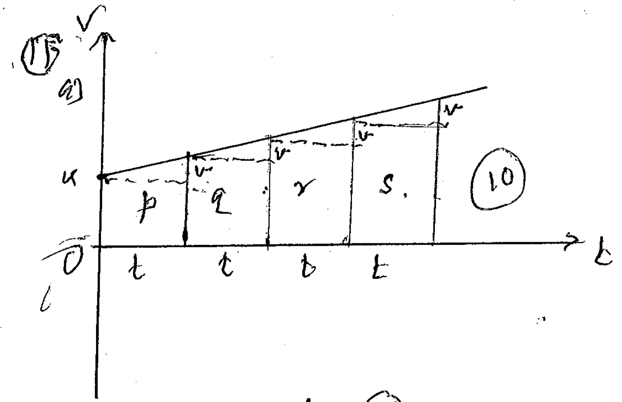
II $P(A) = P(A\cap B) + P(A\cap B')$

$= \frac{22}{100} + \frac{8}{25} = \frac{54}{100}$ (5)

III $P(B) = P(A\cap B) + P(A'\cap B) = \frac{33}{100}$ (5)

IV $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{2}{3}$ (5) $\boxed{25}$

15 a)



$p = \frac{1}{2}(2u+v)t$ (10)

$q = \frac{1}{2}(2u+3v)t$ (10)

$r = \frac{1}{2}(2u+5v)t$ (10)

$\Rightarrow p + r = 2q$

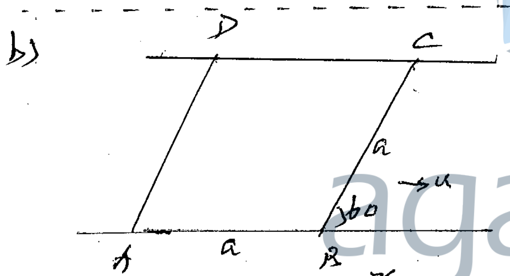
$s = \frac{t}{2}(2u+7v)$ (5)

$2r - q = \frac{t}{2}\left[2(2u+5v) - (2u+3v)\right]$ (10)

$= \frac{t}{2}\left[2u+7v\right]$

$\Rightarrow s = 2r - q$ (5) $\boxed{60}$

b)

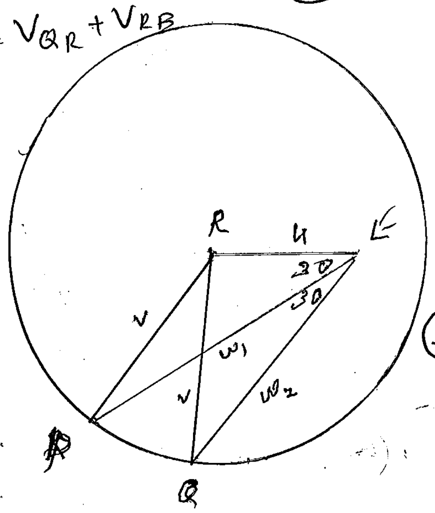


$V_{BE} \xrightarrow{u}$ $\quad V_{PE}$ $\angle 30$ $\quad V_{PB}$ (V)

V_{QE} $\angle 60$ (5) V_{QB} (V)

$V_{PE} = V_{PB} + V_{BE}$ (5)

$V_{QE} = V_{QB} + V_{BE}$



$V_{PE} = w_1 = u\cos 30 + \sqrt{v^2 - u^2\sin^2 30}$ (10)

$= u\frac{\sqrt{3}}{2} + \sqrt{\frac{4v^2 - u^2}{2}}$

$= \frac{1}{2}\left[\sqrt{3}u + \sqrt{4v^2 - u^2}\right]$ (5)

$V_{QE} = w_2 = u\cos 60 + \sqrt{v^2 - u^2\sin^2 60}$ (10)

$= \frac{u}{2} + \sqrt{\frac{4v^2 - 3u^2}{2}}$

$= \frac{1}{2}\left[u + \sqrt{4v^2 - 3u^2}\right]$ (5)

$t_{AC} = t_{AD}$

$\frac{4a\cos 30}{w_1} = \frac{2a}{w_2}$ (10)

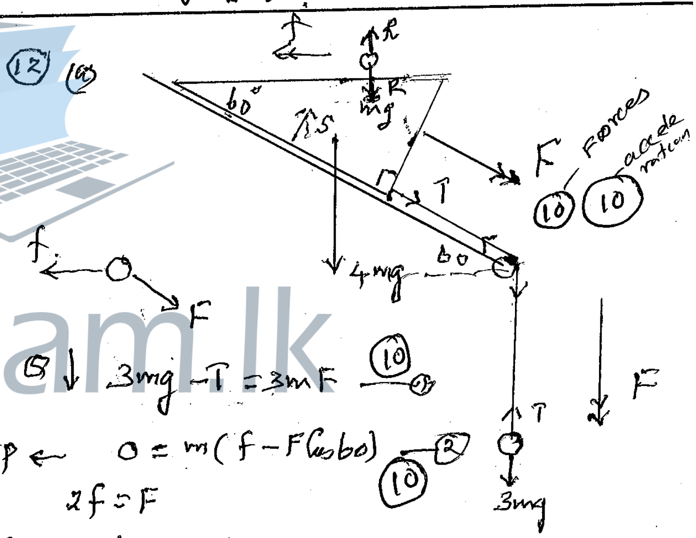
$\sqrt{3}w_2 = w_1$

$\frac{\sqrt{3}}{2}\left(u + \sqrt{4v^2 - 3u^2}\right) = \left(\sqrt{3}u + \sqrt{4v^2 - u^2}\right)\frac{1}{2}$ (5)

$\sqrt{3}\sqrt{4v^2 - 3u^2} = \sqrt{3}u + \sqrt{4v^2 - u^2}$ (5) $\boxed{90}$

$v = u$

12 (a)



$\downarrow$ $3mg - T = 3mF$ (10) ①

$P \leftarrow$ $0 = m(f - F\cos 60)$ (10) ②

$2f = F$

for Wedge + P

$\searrow$ $T + 5mg\sin 60^\circ = 4mF + m(F - f\cos 60)$ (15) ③

①+③ ⇒ $\left(3 + 5\frac{\sqrt{3}}{2}\right)g = 8F - \frac{f}{2}$

$\frac{6 + 5\sqrt{3}}{2}g = \frac{31}{2}f$

$f = \frac{6 + 5\sqrt{3}}{31}g$ (5)

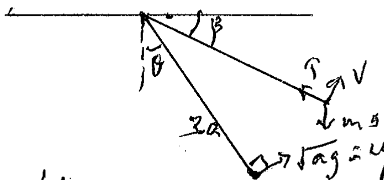
for P $\leftarrow$ $s = ut + \frac{1}{2}at^2$ (10)

$2l = 0 + \frac{1}{2}\left(\frac{6 + 5\sqrt{3}}{31}\right)gt^2$

$t = \sqrt{\frac{124 l}{(6 + 5\sqrt{3})g}}$ (5)

$\boxed{75}$

12 b)



10

using Conservation of Energy.

$\frac{1}{2}mu^2 - mg\,3a\cos\theta = \frac{1}{2}mV^2 - mg\,3a\sin\beta$ 15

$V^2 = 3ag(2\cos\beta - 1)$ 5

↑ $T - mg\sin\beta = m\frac{V^2}{3a}$ 10

$T = mg[3\sin\beta - 1]$ 10

$V=0 \Rightarrow \beta = \pi/6$ 5

$\Rightarrow T > 0$

$\therefore T_{min} = \frac{mg}{2}$ 10 $\beta = \frac{\pi}{6}$

$T_{max} = 2mg$ 10 $\beta = \frac{\pi}{2}$ $\boxed{75}$

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13 (i) $T_0 = 4mg$ 5

$\frac{\lambda}{3a} \times 2a = 4mg$ 5

$\lambda = 6mg$ 5 $\boxed{15}$

(ii)

at P

$4mg - T = 4m\ddot{x}$ 10

$4mg - \frac{6mg}{3a}(2a+x) = 4m\ddot{x}$ 10

$\ddot{x} = -\frac{g}{2a}x$ 5 $\boxed{35}$

(iii)

$x=0$ is centre

$x = A\cos\omega t + B\sin\omega t$

5 $\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$

5 $\ddot{x} = -\omega^2(A\cos\omega t + B\sin\omega t)$

$= -\omega^2 x$

$\therefore \omega = \sqrt{\frac{g}{2a}}$ 5

$t=0,\ x=0,\ \dot{x}=u$ 5

$\Rightarrow$ 5 $A=0$ 5

$u = B\omega \Rightarrow B = \frac{u}{\omega} = u\sqrt{\frac{2a}{g}}$ 5 $\boxed{35}$

(iv)

5 $\therefore x = \frac{u}{\omega}\sin\omega t \Rightarrow \dot{x} = u\cos\omega t$ 5

$x = 3a$, 5 $\dot{x}=0$, $t = T \Rightarrow 0 = u\cos\omega t$ 5

$\omega t = \frac{\pi}{2}$ 5

$\therefore 3a = \frac{u}{\omega}\sin\frac{\pi}{2}$ 5

$u = 3a\omega = 3\sqrt{\frac{ag}{2}}$ 5

amplitude is $3a$ 5 $\boxed{40}$

$t_{BA} = \frac{\pi-\theta}{\omega}$ 5 $\cos\theta = \frac{2}{3}$

$t_{CB} = \frac{\pi}{2\omega}$ 5

Total time $= \frac{\pi}{2\omega} + \frac{\pi-\theta}{\omega}$ 5

$= \frac{1}{\omega}\left(\frac{\pi}{2} + \pi - \theta\right)$

$= \frac{1}{\omega}\left(\frac{3\pi}{2} - \theta\right)$

$= \sqrt{\frac{2a}{g}}\left[\frac{3\pi}{2} - \cos^{-1}\frac{2}{3}\right]$ 10 $\boxed{25}$

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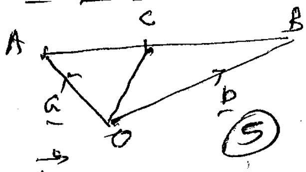
14 (a)



5

$\vec{OC} = \vec{OA} + \vec{AC}$

$\underline{c} = \underline{a} + \lambda(\vec{AB})$ 10

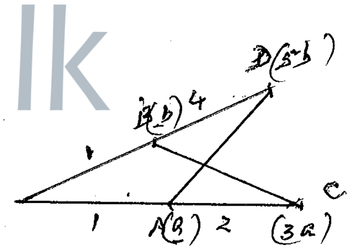
$= \underline{a} + \lambda(\underline{b} - \underline{a})$ 5

$= (1-\lambda)\underline{a} + \lambda\underline{b}$

let $1-\lambda = \alpha$

$\therefore \lambda = 1-\alpha$

$\underline{c} = \alpha\underline{a} + (1-\alpha)\underline{b}$ 5 $\boxed{25}$



Let M is on AD and N is on BC

from the first part $\underline{m} = \lambda\underline{a} + (1-\lambda)5\underline{b}$ 10

$\underline{n} = \mu 3\underline{a} + (1-\mu)\underline{b}$

where λ, μ are parameters 10

to find the intersecting point

$\underline{m} = \underline{n}$ 5

$\Rightarrow \lambda\underline{a} + (1-\lambda)5\underline{b} = \mu 3\underline{a} + (1-\mu)\underline{b}$ 5

$(\lambda - 3\mu)\underline{a} + [5(1-\lambda) - (1-\mu)]\underline{b} = \underline{0}$

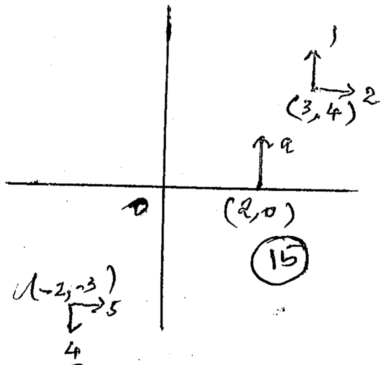
$\Rightarrow \lambda - 3\mu = 0$ 5 $5\lambda - \mu = 4$ 5

$\mu = \frac{2}{7}$, $\lambda = \frac{6}{7}$ 5

$\underline{m} = \frac{6}{7}\underline{a} + \frac{5}{7}\underline{b}$ 5 $\boxed{50}$

M divides AD in 1 : 6

14 b)



$\rightarrow \quad X = 7$ (5)

$\uparrow \quad Y = a - 3$ (5)

O ↺ $a.2 + 1.3 - 2.4 + 5.3 + 4.2 = \pm 24$ (10)

$2a + 18 = \pm 24$

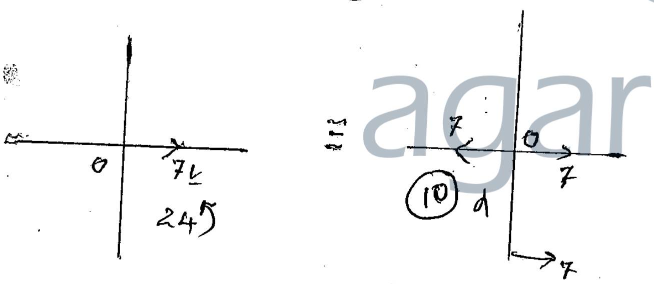
$2a = 6, -42$

$a = 3, -21$ (5) (5)

$a = 3 \Rightarrow \underline{F} = 7\underline{i}$ (5)

$a = -21 \Rightarrow F = 7\underline{i} - 24\underline{j}$ (5)

$a = 3 \Rightarrow G = 24$ ↺ (5)

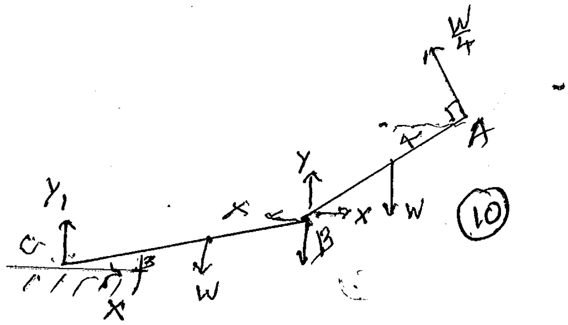


$d = \frac{G}{7} = \frac{24}{7}$

Line Eq^n of the line of action of the equivalent force is $y = -\frac{24}{7}$ (5)

75

15 (a)



for AB ↺ $\frac{W}{4} \cdot 2a - W \cdot a\cos\alpha = 0$ (10)

$\cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$ (5)

$\rightarrow \quad X = \frac{W}{4}\cos 30 = \frac{\sqrt{3}W}{8}$ (5)

$\uparrow \quad Y = W - T\sin 30$ (5)

$= W - \frac{W}{4} \cdot \frac{1}{2}$

$= \frac{7W}{8}$ (5)

for BC ↺

$X.2a\sin\beta + Y.2a\cos\beta - W a\cos\beta = 0$ (10)

$\frac{\sqrt{3}}{8}W.2\sin\beta - \frac{7W}{8}.2\cos\beta - W\cos\beta = 0$ (5)

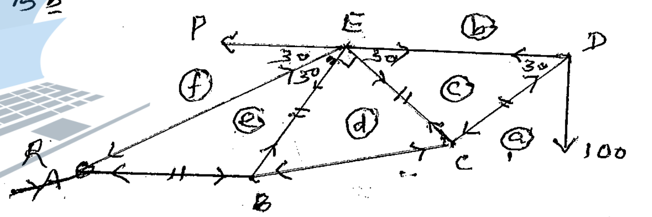
$\sqrt{3}\tan\beta = 11$

$\tan\beta = \frac{11}{\sqrt{3}}$

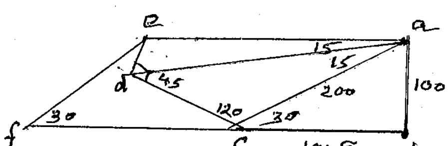
$\beta = \tan^{-1}\left(\frac{11}{\sqrt{3}}\right)$ (5)

60

15 b



Reaction on A is in the direction $\overrightarrow{AB}$ for the equilibrium of System, three forces 100, P, R. Their line of action of these forces meet at D. (10)



faae is a Rhombus. (30)

$\Rightarrow ef = 200$

$\frac{de}{\sin 15} = \frac{ad}{\sin 120} = \frac{200}{\sin 45}$ (10)

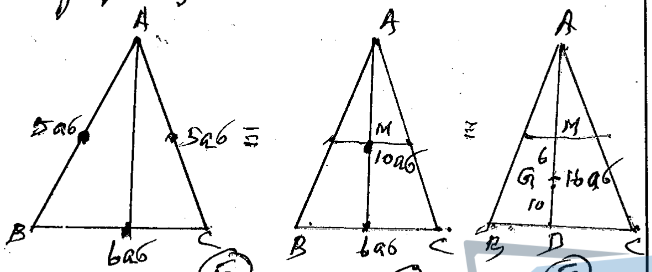
$ad = 100\sqrt{6}$, $de = 100(\sqrt{3}-1) = de$

$ae = 200 = cf$

Rod	Notation	Thrust	Tension	
AB	ae	200	-	5
BC	da	$100\sqrt{6}$	-	5
CD	ca	200	-	5
DE	bc	-	$100\sqrt{3}$	5
EC	cd	-	$100(\sqrt{3}-1)$	5
BE	ed	-	$100(\sqrt{3}-1)$	5
AE	fe	200	-	5

$P = 200 + 100\sqrt{3} = 100(2+\sqrt{3})$ 5 $\boxed{90}$

(16) a) by symmetry c.m lies on median through A



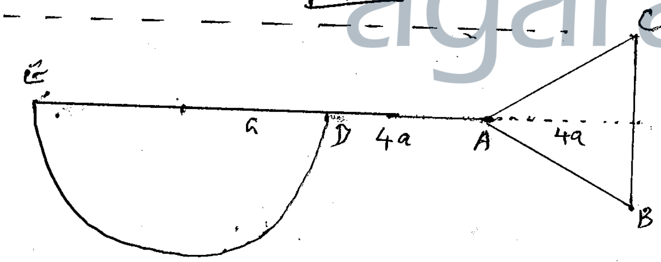
5 5 5

$\frac{MG}{GD} = \frac{6}{10} = \frac{3}{5}$ 5

$MG = \frac{3}{8}\cdot 2a = \frac{3}{4}a$ 5

$AG = 2a + \frac{3}{4}a = \frac{11a}{4}$ 5 $\boxed{30}$

b) Theory $\boxed{40}$



	Mass		C.m from BC		C.m from AD	
Hemisphere	$2\pi a^2\rho$	5	$9a$	5	$\frac{a}{2}$	5
AD	$4a\rho$	5	$6a$	5	0	5
ABC	$16a\rho$	5	$\frac{5a}{4}$	5	0	5
System	$(2\pi a^2+20a)\rho$	5	$\bar{x}$		$\bar{y}$	

$(2\pi a^2+20a)\rho\,\bar{x} = 2\pi a^2\rho\cdot 9a + 4a\rho\cdot 6a + 16a\rho\cdot\frac{5a}{4}$ 10

$\bar{x} = \frac{9\pi a^2+22a}{\pi a+10}$ 5

$(2\pi a^2+20a)\rho\,\bar{y} = 2\pi a^2\rho\cdot\frac{a}{2}$ 10

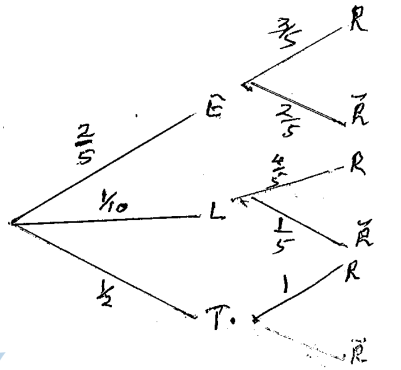
$\bar{y} = \frac{\pi a^2}{2(\pi a+10)}$ 5 $\boxed{80}$

17) a) $P(A/B) = \frac{P(A\cap B)}{P(B)}$ 10 $P(B)>0$

b) A_i are partitions in sample space S; & B is a event.

$P(B) = \sum_i P(A_i)\cdot P(B/A_i)$ 20

c) $R = \{$Student receiving the message$\}$



40

$P(R) = \frac{2}{5}\cdot\frac{3}{5} + \frac{1}{10}\cdot\frac{4}{5} + \frac{1}{2}\cdot 1$ 20

$= \frac{41}{50}$ 10

$P(E/R) = \frac{P(E)\cdot P(R/E)}{P(R)}$ 30

$= \frac{\frac{2}{5}\cdot\frac{3}{5}}{\frac{41}{50}}$ 10

$= \frac{12}{41}$ 10 $\boxed{150}$

1)

For $n=1$

$L.H.S = \frac{1}{1.2} = \frac{1}{2}$ $\quad R.H.S = \frac{1}{1+1} = \frac{1}{2}$

$L.H.S = R.H.S$

The result is true for $n=1$ (5)

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$

$$\sum_{r=1}^{p} \frac{1}{r(r+1)} = \frac{p}{p+1} \quad (5)$$

Now $n=p+1$

$$\sum_{r=1}^{p+1} \frac{1}{r(r+1)} = \frac{p}{(p+1)} + \frac{1}{(p+1)(p+2)} \quad (5)$$

$$= \frac{p(p+2)+1}{(p+1)(p+2)}$$

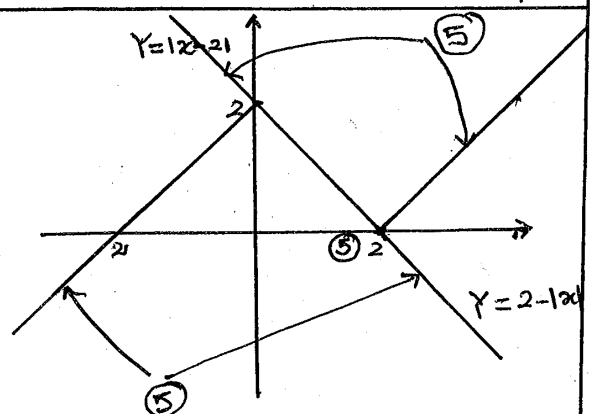
$$= \frac{(p+1)^2}{(p+1)(p+2)} = \frac{p+1}{p+2} \quad (5)$$

hence if the result is true for $n=p$ then it is also true for $n=p+1$ (5)

hence by the Principle of mathematical Induction the result is true for all $n \in \mathbb{Z}^+$

$\boxed{25}$

2)

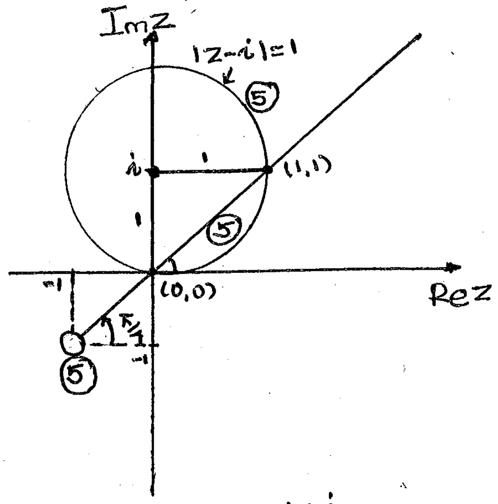


$|x-2| = 2-|x| \iff 0 \le x \le 2$ (5)

$|x-2|+|x| = 2 \iff 0 \le x \le 2$

$\therefore$ The solutions of x are $0 \le x \le 2$ (5) $\boxed{25}$

3)



$\therefore$ $z = 0$ (5) or $z = 1+i$ (5)

$\boxed{25}$

4)

$$\lim_{x \to 2} \frac{\cos \frac{\pi x}{4}}{\sqrt{x+3} - \sqrt{5}} \quad (5)$$

$$= \lim_{x \to 2} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi x}{4}\right)\left(\sqrt{x+3}+\sqrt{5}\right)}{(x+3)-5} \quad (5)$$

$$= \lim_{x \to 2} \frac{\sin \frac{\pi}{4}(2-x)\left(\sqrt{x+3}+\sqrt{5}\right)}{(x-2)}$$

$$= -\frac{\pi}{4} \lim_{x \to 2} \frac{\sin \frac{\pi}{4}(x-2)}{\frac{\pi}{4}(x-2)} \ (5) \ . \lim_{x \to 2}\left(\sqrt{x+3}+\sqrt{5}\right)$$

$\downarrow$ (5)

$$= -\frac{\pi}{4} . 1 . 2\sqrt{5}$$

$$= -\frac{\sqrt{5}\pi}{2} \quad (5)$$

$\boxed{25}$

Page1

5. $x = 2(1-\cos\theta)$

$\frac{dx}{d\theta} = 2\sin\theta = 4\sin\theta/_2 \cos\theta/_2$

$y = 2(\theta + \sin\theta)$

$\frac{dy}{d\theta} = 2(1+\cos\theta) = 4\cos^2\theta/_2$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{4\cos^2\theta/_2}{4\sin\theta/_2 \cos\theta/_2}$ (5)

$\frac{dy}{dx} = \cot\theta/_2$ (5)

Point $(4, 2\pi)$ on $c \Rightarrow \theta = \pi$ (5)

$\left(\frac{dy}{dx}\right)_{\theta=\pi} = 0$ (5)

Equation of the tangent is

$y = 2\pi$ (5)

$\boxed{25}$

6. $\frac{d}{dx}\left\{x\sqrt{1-x^2}\right\}$

$= x \frac{1\,(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2}$ (10)

$= \frac{1-2x^2}{\sqrt{1-x^2}}$ (5)

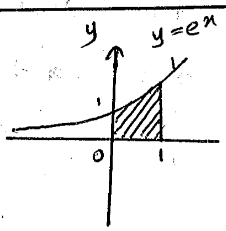
$\int \frac{1+x^2}{\sqrt{1-x^2}}dx = -\frac{1}{2}\int \frac{1-2x^2}{\sqrt{1-x^2}}dx + \int \frac{\frac{3}{2}}{\sqrt{1-x^2}}dx$ (5)

$= -\frac{1}{2}x\sqrt{1-x^2} + \frac{3}{2}\sin^{-1}x + c$

(5) $\boxed{25}$

7.

The volume generated



$= \int_0^1 \pi (e^x)^2 dx$ (10)

$= \int_0^1 \pi e^{2x} dx$ (5)

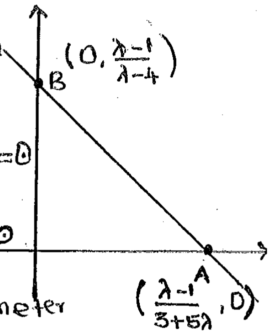
$= \pi \left[\frac{e^{2x}}{2}\right]_0^1$ (5)

$= \frac{\pi}{2}(e^2 - 1)$ (5)

$\boxed{25}$

page 2

8) The required equation can be writen



$(3x - 4y + 1) + \lambda(5x + y - 1) = 0$ (5)

$(3+5\lambda)x + (\lambda - 4)y + (1-\lambda) = 0$

λ - parameter

But $\left|\frac{\lambda-1}{\lambda-4}\right| = \left|\frac{\lambda-1}{3+5\lambda}\right|$ (5)

$\lambda - 1 = 0$ or $3 + 5\lambda = \pm(\lambda - 4) \Rightarrow \lambda = -\frac{7}{4}$ or $\frac{1}{6}$

The equations (5) or $\lambda =$

$23x + 23y - 11 = 0$

$23x - 23y + 5 = 0$

$8x - 3y = 0$ (10)

$\boxed{25}$

9) $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$(0,0) \Rightarrow c = 0$

$(0,2) \Rightarrow 0 + 4 + 0 + 4f + c = 0 \Rightarrow f = -1$ (5)

$S \equiv x^2 + y^2 + 2gx - 2y = 0$

$S' \equiv x^2 + y^2 - 2x + 4y - 6 = 0$

Centre $O' \equiv (1, -2)$ (5)

equation of common chord is

$S - S' = 0$

$(2g + 2)x - 6y + 6 = 0$ (5)

$(1,-2) \Rightarrow 2(g+1) + 12 + 6 = 0 \Rightarrow g = -10$ (5)

$S \equiv x^2 + y^2 - 20x - 2y = 0$ (5) $\boxed{25}$

10) $\tan^{-1}\left(\frac{x}{x-1}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

$\Downarrow$ A, $\Downarrow$ B, $\Downarrow$ C

$A + B = C$ (5) $\Rightarrow \tan(A+B) = \tan C$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan C$ (5)

$\frac{\frac{x}{x-1} + \frac{x+2}{x+3}}{1 - \frac{x(x+2)}{(x-1)(x+3)}} = \frac{2}{3}$ (5)

$\frac{2x^2 + 4x - 2}{-3} = \frac{2}{3}$ (5)

$x^2 + 2x - 1 = -1$

$x = 0$ or $x = -2$ (5)

$\boxed{25}$

11 (a)

Suppose that 1 is a root of $(p+1)^2x^2+8x+2(p+1)=0$.

By substituting $x=1$, we must have $(p+1)^2+8+2(p+1)=0$ (5)

This is impossible, as $p>-1$ implies that $(p+1)^2+8+2(p+1)>0$ (5)

$\therefore$ 1 is not a root of $(p+1)^2x^2+8x+2(p+1)=0$ $\boxed{10}$

The discriminant

$\Delta = 8^2-4(p+1)^2\,2(p+1)$ (10)

$= 8\{8-(p+1)^3\} \geq 0$ $(\because -1<p\leq 1)$ (5)

$\therefore$ α and β are both real (5) $\boxed{20}$

$\alpha+\beta = -\dfrac{8}{(p+1)^2}$ (5), and $\alpha\beta = \dfrac{2}{p+1}$ (5)

$$\frac{1}{(\alpha-1)(\beta-1)} = \frac{1}{\alpha\beta-(\alpha+\beta)+1} \quad (5)$$

$$= \frac{1}{\frac{2}{p+1}+\frac{8}{(p+1)^2}+1}$$

$$= \frac{(p+1)^2}{p^2+4p+11} \quad (5) \quad \boxed{20}$$

$$\frac{\alpha}{\alpha-1}+\frac{\beta}{\beta-1} = \frac{2\alpha\beta-(\alpha+\beta)}{(\alpha-1)(\beta-1)} \quad (5)$$

$$= \left(\frac{4}{p+1}+\frac{8}{(p+1)^2}\right)\cdot\frac{(p+1)^2}{p^2+4p+11}$$

$$= \frac{4(p+3)}{p^2+4p+11} \quad (5)$$

(5)

$$\frac{\alpha}{\alpha-1}\cdot\frac{\beta}{\beta-1}$$

$$= \frac{\alpha\beta}{(\alpha-1)(\beta-1)}$$

$$= \frac{2}{p+1}\cdot\frac{(p+1)^2}{p^2+4p+11} \quad (5)$$

$$= \frac{2(p+1)}{p^2+4p+11} \quad (5)$$

Hence, the required quadratic equation is given by

$$x^2-\frac{4(p+3)}{p^2+4p+11}x+\frac{2(p+1)}{p^2+4p+11}=0 \quad (5)$$

$(p^2+4p+11)x^2-4(p+3)x+2(p+1)=0$ ——(*) $\boxed{30}$

$$\frac{\alpha}{\alpha-1}+\frac{\beta}{\beta-1} = \frac{4(p+3)}{(p+2)^2+7} > 0 \quad (5) \quad (\because p>-1)$$

$$\frac{\alpha}{\alpha-1}\cdot\frac{\beta}{\beta-1} = \frac{2(p+1)}{(p+2)^2+7} > 0 \quad (5) \quad (\because p>-1)$$

Hence, both of these roots are positive $\boxed{10}$

(b) $ax^n+b = (x^2-1)\phi(x)+x+2$ (10)

$x=1 \Rightarrow a+b=3$ (5)

$x=-1 \Rightarrow -a+b=1$ (5)

$a=1,\ b=2$ (5)

$x^n+2 \equiv (x^2-1)\phi(x)+x+2$

$n=7 \Rightarrow x^7+2 \equiv (x^2-1)\phi_1(x)+x+2$ (5) —①

$n=5 \Rightarrow x^5+2 \equiv (x^2-1)\phi_2(x)+x+2$ (5) —②

$n=3 \Rightarrow x^3+2 \equiv (x^2-1)\phi_3(x)+x+2$ (5) —③

①+②+③ $\Rightarrow$

$x^7+x^5+x^3+6 \equiv (x^2-1)[\phi_1(x)+\phi_2(x)+\phi_3(x)] + 3x+6$ (5)

$\therefore$ Remainder $= 3x+6$ (5) $\boxed{50}$

$1-\dfrac{1}{\alpha} = \dfrac{\alpha-1}{\alpha}$, $1-\dfrac{1}{\beta} = \dfrac{\beta-1}{\beta}$

Replacing x by $\frac{1}{x}$ in (*), we get (5)

$(p^2+4p+11)\left(\frac{1}{x}\right)^2-4(p+3)\frac{1}{x}+2(p+1)=0$

$2(p+1)x^2-4(p+3)x+p^2+4p+11=0$ (5) $\boxed{10}$

12

(a)

(i) $^9C_3 = \frac{9!}{6!\times 3!}$ (10)

$= 84$ (5)

(ii) $^{18}C_3 - {}^9C_3$ (10)

$= 816 - 84$

$= 732$ (5)

(iii) $^6C_1\,^6C_1\,^6C_1 = 216$ (5) (10)

(iv) $^3C_1\,^2C_1\,^1C_1\times 3 = 18$ (5) (15) (5)

$\boxed{70}$

(b)

$U_r = \frac{r+3}{r(r+1)(r+2)}$, $V_r = \frac{2r+3}{r(r+1)}$

$V_r - V_{r+1}$

$= \frac{2r+3}{r(r+1)} - \frac{2r+5}{(r+1)(r+2)}$ (5)

$= \frac{(2r+3)(r+2)-(2r+5)r}{r(r+1)(r+2)}$ (5)

$= \frac{2r^2+7r+6-2r^2-5r}{r(r+1)(r+2)}$

$= \frac{2(r+3)}{r(r+1)(r+2)}$ (5)

$= 2U_r$ (5)

$\boxed{20}$

$2U_r = V_r - V_{r+1}$

$r=1;\ 2U_1 = V_1 - \cancel{V_2}$ (5)

$r=2;\ 2U_2 = \cancel{V_2} - \cancel{V_3}$

$\vdots$

$r=n-1;\ 2U_{n-1} = \cancel{V_{n-1}} - \cancel{V_n}$ (5)

$r=n;\ 2U_n = \cancel{V_n} - V_{n+1}$

$2\sum_{r=1}^{n} U_r = V_1 - V_{n+1}$ (5)

$= \frac{5}{2} - \frac{2n+5}{(n+1)(n+2)}$ (5)

$\Rightarrow \sum_{r=1}^{n} U_r = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}$ (5) $\boxed{25}$

$\lim_{n\to\infty}\sum_{r=1}^{n} U_r = \lim_{n\to\infty}\left\{\frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}\right\}$ (5)

$= \lim_{n\to\infty}\left\{\frac{5}{4} - \frac{\frac{2}{n}+\frac{5}{n^2}}{2(1+\frac{1}{n})(1+\frac{2}{n})}\right\}$ (5)

$= \frac{5}{4} - 0$

$= \frac{5}{4}$ (5)

$\therefore \sum_{r=1}^{\infty} U_r$ is convergent and the sum is $\frac{5}{4}$ (5) $\boxed{20}$

$\sum_{r=3}^{\infty} 3U_r$

$= 3\sum_{r=1}^{\infty} U_r - 3U_1 - 3U_2$ (5)

$= 3\times\frac{5}{4} - 3\left(\frac{2}{3}\right) - 3\left(\frac{5}{24}\right)$ (5)

$= \frac{9}{8}$ // (5)

$\boxed{15}$

page 4

13

$z \in \mathbb{C}$ $\quad z = x+iy \quad x, y \in \mathbb{R}$

a) $|z| = \sqrt{x^2+y^2}$ (5) $\quad \bar{z} = x-iy$ (5)

1. $z.\bar{z} = (x+iy)(x-iy) = x^2 - (iy)^2$ (5)

$= x^2+y^2 = |z|^2$ (5)

$z+\bar{z} = x+iy+x-iy = 2x = 2\operatorname{Re} z$ (5)

$z-\bar{z} = x+iy-x+iy = 2iy = 2i\operatorname{Im} z$ (5)

$\boxed{30}$

b) $z = x+iy = \sqrt{x^2+y^2}\left(\frac{x}{\sqrt{x^2+y^2}} + i\frac{y}{\sqrt{x^2+y^2}}\right)$ (5)

$= |z|(\cos\theta + i\sin\theta)$

(5)

$|z| = \sqrt{x^2+y^2} \quad \operatorname{Arg}(z) = \theta$

$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$

$z_1 = r_1(\cos\theta_1+i\sin\theta_1) \quad |z_1| = r_1$

$\operatorname{Arg}(z_1) = \theta_1$

$z_2 = r_2(\cos\theta_2+i\sin\theta_2) \quad |z_2| = r_2$

$\operatorname{Arg}(z_2) = \theta_2$

$z_1.z_2 = r_1.r_2(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)$

$= r_1r_2\left((\cos\theta_1\cos\theta_2+i^2\sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)\right)$ (5)

$= r_1r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$ (5)

$|z_1.z_2| = r_1r_2 = |z_1||z_2|$ (5)

$\arg(z_1z_2) = \theta_1+\theta_2 = \arg(z_1)+\arg(z_2)$ (5)

$\boxed{30}$

c) $z_1 = \frac{4}{1-i\sqrt{3}} \quad z_2 = \frac{2}{1+i}$

$z_1 = \frac{4(1+i\sqrt{3})}{(1-i^2 3)}$ (5) $\quad z_2 = \frac{2(1-i)}{1-i^2}$ (5)

$z_1 = (1+i\sqrt{3}) \quad z_2 = 1-i$

$z_1 = 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$ (5) $\quad z_2 = \sqrt{2}\left(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$ (5)

$z_1 = 2(\cos\pi/3 + i\sin\pi/3)$ (5)

$|z_1| = 2$ (5)

$\operatorname{Arg}(z_1) = \pi/3$

$z_2 = \sqrt{2}(\cos(-\pi/4)+i\sin(-\pi/4))$ (5)

$|z_2| = \sqrt{2}$ (5)

$\operatorname{Arg}(z_2) = -\pi/4$

$|z_1.z_2| = |z_1|.|z_2| = 2\sqrt{2}$ (5)

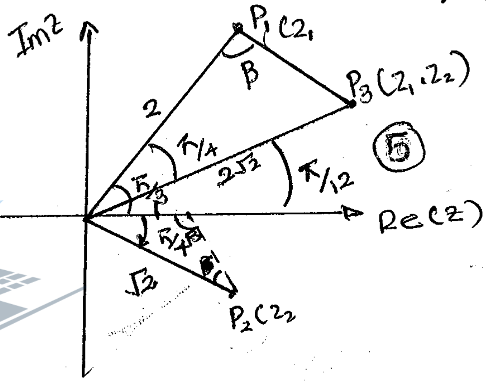
$\arg(z_1.z_2) = \arg(z_1)+\arg(z_2)$

$= \pi/3 - \pi/4 = \frac{\pi}{12}$ (5)

$\operatorname{Arg}(z_1z_2) = \pi/12$

$z_1.z_2 = |z_1z_2|(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$

$= 2\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$ (5)



(5)

$\boxed{60}$

c) $z_1^3+2z_2^4 = 2^3(\cos 3\times\pi/3 + i\sin 3\times\pi/3)$

$+ 2\times(\sqrt{2})^4(\cos(4\times -\pi/4) + i\sin 4\times -\pi/4)$ (5)

$= 2^3[\cos\pi + i\sin\pi + \cos\pi - i\sin\pi]$ (5)

$= 2^3 \times 2.\cos\pi$ (5)

$= -16$ (5)

$\boxed{30}$

Page 5

14. $f(x) = \dfrac{x+1}{(x+3)^2}$

$$f'(x) = \frac{(x+3)^2(1) - (x+1)2(x+3)}{(x+3)^4} \quad 10$$

$$= \frac{(x+3)[x+3-2x-2]}{(x+3)^4}$$

$$= -\frac{x-1}{(x+3)^3}$$

$$f''(x) = -\left\{\frac{(x+3)^3(1) - (x-1)3(x+3)^2}{(x+3)^6}\right\} \quad 10$$

$$= -\left\{\frac{x+3-3x+3}{(x+3)^4}\right\}$$

$$= \frac{2(x-3)}{(x+3)^4} \qquad \boxed{20}$$

When $x = 0$, $y = \frac{1}{9}$

when $y = 0$, $x = -1$

$$\lim_{x \to -3} f(x) = \infty$$

Vertical asymptote : $x = -3$ 5

$$\lim_{x \to \pm\infty} \frac{x+1}{(x+3)^2} = \lim_{x \to \pm\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{\left(1+\frac{3}{x}\right)^2} = 0$$

Horizontal asymptote : $y = 0$ 5

when $f'(x) = 0$, $x = 1$ 5

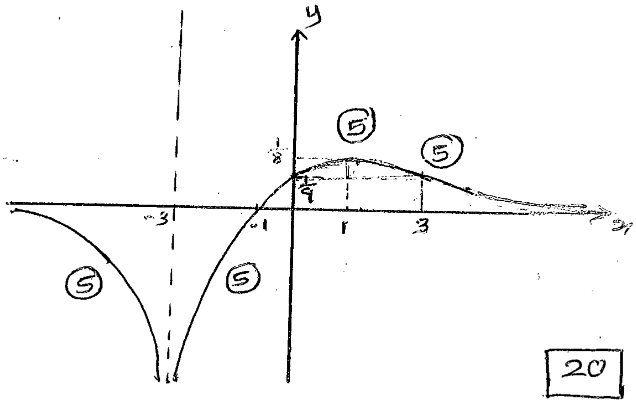
$x < -3$	$-3 < x < 1$	$x > 1$
$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$
decreasing 5	increasing 5	decreasing 5

$(1, \frac{1}{8})$ is a local maximum 5

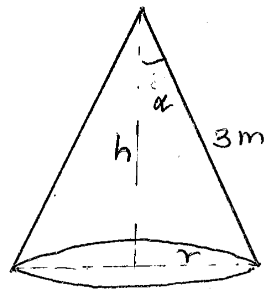
$f''(x) = 0 \Leftrightarrow x = 3$ 5

$x < -3$	$-3 < x < 3$	$x > 3$
$f''(x) < 0$	$f''(x) < 0$	$f''(x) > 0$
concave down 5	concave down 5	concave up 5

$(3, \frac{1}{9})$ is a point of inflection 5 $\boxed{60}$



$\boxed{20}$



$h^2 + r^2 = 3^2$ 5

$\Rightarrow r^2 = 9 - h^2$

Volume $V = \frac{1}{3}\pi r^2 h$ 5

$= \frac{1}{3}\pi h(9 - h^2)$ 5

$$\frac{dV}{dh} = \frac{1}{3}\pi(9 - 3h^2) \quad 10$$

$$= -\pi(h^2 - 3)$$

$\dfrac{dV}{dh} = 0 \Leftrightarrow h = \sqrt{3}$ $[\because h > 0]$ 5

For $0 < h < \sqrt{3}$, $\dfrac{dV}{dh} > 0$ 5 and $h > \sqrt{3}$, $\dfrac{dV}{dh} < 0$ 5

$\therefore$ V is maximum when $h = \sqrt{3}$ 5

$h = \sqrt{3} \Rightarrow r = \sqrt{6}$

$\tan\alpha = \dfrac{r}{h} = \dfrac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$

$\therefore$ V is maximum when $\alpha = \tan^{-1}(\sqrt{2})$ 5 $\boxed{50}$

Page 6

15

a) I $\dfrac{d\ln(x^2+1)}{dx} = \dfrac{1}{(x^2+1)} \times 2x = \dfrac{2x}{(x^2+1)}$ 5

$\int$ing both side w.r.t x

$$\int \frac{2x}{x^2+1} = \ln(x^2+1) + c' \quad 5 \quad 5$$

$$\int \frac{x}{x^2+1} = \frac{\ln(x^2+1)}{2} + C$$

C - arbitary constant $\boxed{15}$

II $\dfrac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} = \dfrac{Ax+B}{(x^2+1)} + \dfrac{C}{(x-2)} + \dfrac{D}{(x+2)}$ 10

$x^3+4x^2-4x+4 = (Ax+B)(x^2-4) + C(x^2+1)(x+2) + D(x^2+1)(x-2)$

$x=2 \quad 20C = 20 \Rightarrow C=1$

$x=-2 \quad -20D = 20 \Rightarrow D=-1$

con $\quad 2C-2D-4B = 4 \Rightarrow B=0$

$x^3 \quad A+C+D = 1 \quad \Rightarrow A=1$

4 eq^n / ans 15

$$\frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} = \frac{x}{x^2+1} + \frac{1}{x-2} + \frac{-1}{x+2}$$

$\therefore \int$ing both side w.r.t x

$$\int \frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)}dx = \int\frac{x\,dx}{x^2+1} + \int\frac{dx}{x-2} - \int\frac{dx}{x+2}$$

5 5 5 5

$$= \frac{\ln(x^2+1)}{2} + \ln|x-2| - \ln|x+2| + K$$

K - arbitary constant $\boxed{45}$

b) $\int x\sin^2 x\,dx = \int \dfrac{x(1-\cos 2x)}{2}dx$ 5

$$= \int \frac{x}{2}dx - \int \frac{x\cos 2x}{2}dx$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} + \int \frac{\sin 2x}{4}dx + c' \quad 5 \; 5 \; 5$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c \quad 5 \; 5$$

c - arbitary con

$\int x\cos^2 x\,dx + \int x\sin^2 x\,dx = \int x(\sin^2 x + \cos^2 x)$ 5

$\Rightarrow \int x\cos^2 x\,dx = \int x\,dx - \int x\sin^2 x\,dx.$ 5

$\Rightarrow \int x\cos^2 x\,dx = \dfrac{x^2}{2} - \left(\dfrac{x^2}{4} - \dfrac{x\sin 2x}{4} - \dfrac{\cos 2x}{8}\right) + c$ 5

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + c \quad 5$$

$\boxed{50}$

c) $x = 1 + 3\sin^2\theta$

$x : 1 \to 4 \qquad x=1 \Leftrightarrow \sin^2\theta = 0, \; \theta = 0$ 5

$\theta : 0 \to \pi/2 \qquad x=4 \Leftrightarrow \sin^2\theta = 1, \; \theta = \pi/2$ 5

$dx = 6\sin\theta\cos\theta\,d\theta \; (0 \le \theta \le \pi/2)$ 5

$$\int_1^4 \frac{dx}{\sqrt{(x-1)(4-x)}} = \int_0^{\pi/2} \frac{6\sin\theta\cos\theta\,d\theta}{\sqrt{3\sin^2\theta \cdot 3\cos^2\theta}} \quad 5$$

$$= \int_0^{\pi/2} \frac{6\sin\theta\cos\theta\,d\theta}{3\sin\theta\cos\theta} \quad 5 \quad = 2\int_0^{\pi/2} d\theta \quad 5$$

$$= 2\,\theta\Big|_0^{\pi/2} \quad 5$$

$$= 2(\pi/2 - 0) = \pi \quad 5$$

$\boxed{40}$

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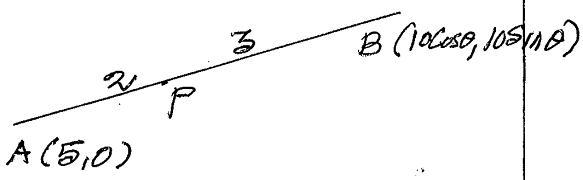
Q16]



$$P \equiv \left(\frac{20\cos\theta + 15}{5}, \frac{20\sin\theta + 0}{5}\right) \quad (10)$$

$P \equiv (4\cos\theta + 3, \ 4\sin\theta)$

$P \equiv (\bar{x}, \bar{y})$ (say)

$\bar{x} = 4\cos\theta + 3 \quad \bar{y} = 4\sin\theta$ (5)

but $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{\bar{x}-3}{4}\right)^2 + \left(\frac{\bar{y}}{4}\right)^2 = 1 \quad (5)$$

$\bar{x}^2 + \bar{y}^2 - 6\bar{x} - 7 = 0$ (5)

put $\bar{x} \to x$, $\bar{y} \to y$

$S \equiv x^2 + y^2 - 6x - 7 = 0$ (5)

$(x-3)^2 + (y-0)^2 = 4^2$ The path is circle (5)

Centre $C \equiv (3, 0)$

radius $= 4$ (5) $\boxed{40}$

$Q \equiv (2, 5)$

$CQ = \sqrt{(3-2)^2 + (0-5)^2}$ (5)

$= \sqrt{26} > 4$ (5)

The point Q lies outside the circle

The equation of tangent is given by

$y - 5 = m(x-2)$ (5)

$mx - y + (5 - 2m) = 0$

$$4 = \left|\frac{m \times 3 - 0 + 5 - 2m}{\sqrt{m^2 + (-1)^2}}\right| \quad (10)$$

$16(m^2+1) = (m+5)^2$

$15m^2 - 10m + 9 = 0$ (5)

$$m = \frac{10 \pm \sqrt{100 - 4(15)(-9)}}{2 \times 15} \quad (10)$$

$$= \frac{5 \pm 4\sqrt{10}}{15} \quad (5)$$

Tangents are

$$y - 5 = \left(\frac{5 \pm 4\sqrt{10}}{15}\right)(x-2) \quad (10)$$

$\boxed{55}$

CD

$2x + 5y - 3(2+x) - 7 = 0$

$x - 5y + 13 = 0$ (10)

S' can be write

$x^2 + y^2 - 6x - 7 + \lambda(x - 5y + 13) = 0$ (5)

$(1,2) \Rightarrow$

$1 + 4 - 6 - 7 + \lambda(1 - 10 + 13) = 0$

$\lambda = 2$ (5)

$S' \equiv x^2 + y^2 - 4x - 10y + 19 = 0$ (5) $\boxed{25}$

$S'' \equiv x^2 + y^2 + 2g''x + 2f''y + c'' = 0$ (say)

$(0,6) \Rightarrow 0 + 36 + 12f'' + c'' = 0$ (5)

$c'' = -36 - 12f''$ (1)

The circles $S' = 0$, $S'' = 0$ are interset orthogonal

$2\{(g'')(-2) + (f'')(-5)\} = c'' + 19$ (10)

$-4g'' - 10f'' = -36 - 12f'' + 19$ (by (1))

$4g'' - 2f'' - 17 = 0$ (5)

put $-g'' = x$, $-f'' = y$ (5)

$4x - 2y + 17 = 0$ (5)

$\boxed{30}$

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Q17]

a) $\sin(A+B) = \sin A\cos B + \cos A\sin B$ —— ① (5)

$\sin(A-B) = \sin\{A+(-B)\}$

$= \sin A\cos(-B) + \cos A\sin(-B)$

$= \sin A\cos B - \cos A\sin B$ (5) —— ②

① + ② ⇒

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$ (5)

put $B \to A$

$\sin(2A) + \sin(A-A) = 2\sin A\cos A$

$\sin 2A = 2\sin A\cos A$ (5) $\boxed{20}$

$\sin\theta\{8\cos\theta\cos2\theta\cos3\theta - 1\}$

$= 8\sin\theta\cos\theta\cos2\theta\cos3\theta - \sin\theta$ (5)

$= 4\sin2\theta\cos2\theta\cos3\theta - \sin\theta$ (5)

$= 2\sin4\theta\cos3\theta - \sin\theta$ (5)

$= \sin7\theta + \sin\theta - \sin\theta$ (5)

$= \sin7\theta$

$\cos\theta\cos2\theta\cos3\theta = \frac{1}{4}$

$\frac{1}{8}\left\{\frac{\sin7\theta}{\sin\theta} + 1\right\} = \frac{1}{4}$ (5)

$\sin7\theta = \sin\theta$

$7\theta = n\pi + (-1)^n\theta$ (5); $n \in \mathbb{Z}$

$n = 0 \Rightarrow \theta = 0$

$n = 1 \Rightarrow \theta = \frac{\pi}{8}$

$n = 2 \Rightarrow \theta = \frac{\pi}{4}$

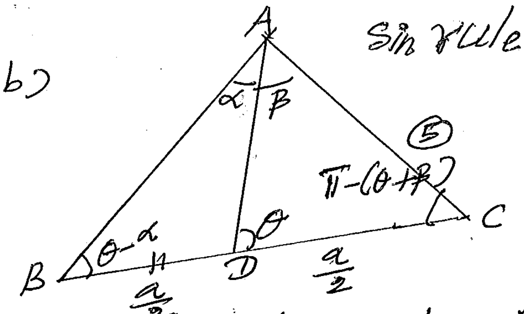
$n = 3 \Rightarrow \theta = \frac{3\pi}{8}$

$n = 4 \Rightarrow \theta = \frac{2\pi}{3} > \frac{\pi}{2}$

Sol^n:- $\left\{\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}\right\}$ (10)

Page - 9 $\boxed{40}$

b)



Sin rule — (5)

for the triangle ABD

$\frac{BD}{\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$ (5)

$\frac{\frac{a}{2}}{\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$

$\frac{a}{2\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$ (5) ①

In △ADC

$\frac{AD}{\sin\{\pi-(\theta+\beta)\}} = \frac{\frac{a}{2}}{\sin\beta}$ (10)

$\frac{AD}{\sin(\theta+\beta)} = \frac{a}{2\sin\beta}$ —— ②

①, ② ⇒

$\frac{a\sin(\theta-\alpha)}{2\sin\alpha} = \frac{a\sin(\theta+\beta)}{2\sin\beta}$ (5)

$\sin\beta\{\sin\theta\cos\alpha - \cos\theta\sin\alpha\} = \sin\alpha\{\sin\theta\cos\beta + \cos\theta\sin\beta\}$ (10)

divide by $\sin\alpha\sin\beta\sin\theta$ both side

$\cot\alpha - \cot\theta = \cot\beta + \cot\theta$ (5)

$\cot\alpha - \cot\beta = 2\cot\theta$ (5) $\boxed{55}$

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c) $2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{6}{5}\right) = \frac{\pi}{2}$ —— ①

$\alpha = \tan^{-1}\left(\frac{1}{5}\right)$ $\beta = \tan^{-1}\left(\frac{6}{5}\right)$

$\Leftrightarrow 2\alpha + \beta = \frac{\pi}{2}$ (5)

$\Leftrightarrow 2\alpha = \frac{\pi}{2} - \beta$

$\Leftrightarrow \tan2\alpha = \tan\left(\frac{\pi}{2} - \beta\right)$ (5)

$\left(2\alpha, \left(\frac{\pi}{2} - \beta\right) \text{ are acute}\right)$

$\tan2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$ (5)

$= \frac{2\left(\frac{1}{5}\right)}{1-\frac{1}{25}}$

$= \frac{\frac{10}{24}}{}$

$= \frac{5}{6}$ (5)

$\tan\left(\frac{\pi}{2} - \beta\right) = \cot\beta$ (5)

$= \frac{5}{6}$

$\therefore 2\alpha + \beta = \frac{\pi}{2}$

$2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{6}{5}\right) = \frac{\pi}{2}$

$\tan^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)$ (5)

$\sin^{-1}\left(\frac{1}{\sqrt{26}}\right) = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)$ (5)

1, $\sqrt{26}$, 5

$\frac{1}{\sqrt{26}} = \sin\left\{\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)\right\}$ $\boxed{35}$