G.C.E A/L Examination March - 2019

Fied Work Centre

FWC

Grade - 13 (2019) **Combined Mathematics - I** **Marking Scheme**

Q1

For $n=1$

$L.H.S = \sum_{r=1}^{1} r2^{r-1} = 1.2^0 = 1$

$R.H.S = 1+(1-1)2^1 = 1$

The result is true for $n=1$ (5)

take any $p \in Z^+$ and assume that the result is true for $n=p$.

i.e $\sum_{r=1}^{p} r2^{r-1} = 1+(p-1)2^p$ (5)

For $n=p+1$

$\sum_{r=1}^{p+1} r2^{r-1} = \sum_{r=1}^{p} r2^{r-1} + (p+1)2^p$ (5)

$= 1+(p-1)2^p + (p+1)2^p$

$= 1+2p.2^p$

$= 1+p.2^{p+1}$ (5)

Hence if the result is true for $n=p$ then it is also true for $n=p+1$ (5)

Hence by the principle of mathematical Induction, the result is true for all $n \in Z^+$

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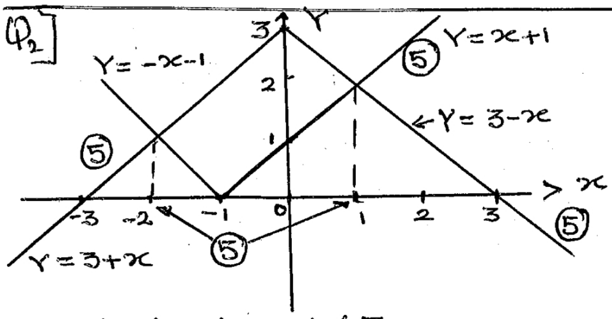
Q2



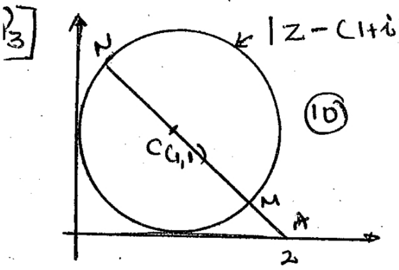
$|x| + |x+1| \leq 3$

$|x+1| \leq 3-|x|$ (5)

The value of x satisfying $-2 \leq x \leq 1$

25

Q3



(10)

$AC = \sqrt{(2-1)^2+(0-1)^2}$

$= \sqrt{2}$ (5)

$|z-2|_{min} = AM = AC - CM = \sqrt{2}-1$ (5)

$|z-2|_{max} = AN = AC + CN = \sqrt{2}+1$ (5)

Q4

$\lim_{x \to \beta} \frac{\tan^2 x - \tan^2\beta}{\sqrt{x}-\sqrt{\beta}}$

$= \lim_{x \to \beta} \frac{(\tan x - \tan\beta)(\tan x + \tan\beta)(\sqrt{x}+\sqrt{\beta})}{(x-\beta)}$ (10)

$= \lim_{x \to \beta} \frac{\left(\frac{\sin x}{\cos x} - \frac{\sin\beta}{\cos\beta}\right)(\tan x + \tan\beta)(\sqrt{x}+\sqrt{\beta})}{(x-\beta)}$ (5)

$= \lim_{x-\beta \to 0} \frac{\sin(x-\beta)}{(x-\beta)} \lim_{x \to \beta} \frac{(\tan x + \tan\beta)(\sqrt{x}+\sqrt{\beta})}{\cos x \cos\beta}$ (5)

$= 1. \frac{(\tan\beta + \tan\beta)(\sqrt{\beta}+\sqrt{\beta})}{\cos\beta.\cos\beta}$ (5)

$= 4\sqrt{\beta} \tan\beta \sec^2\beta$

25

Q5

$x = 2+\cos 4\theta$ $\quad Y = 4\sin 2\theta$

$\frac{dx}{d\theta} = -\sin 4\theta . 4$ (5) $\quad \frac{dY}{d\theta} = 4\cos 2\theta . 2$

$= 8\cos 2\theta$ (5)

$\frac{dY}{dx} = \frac{dY}{d\theta} . \frac{d\theta}{dx} = \frac{8\cos 2\theta}{-\sin 4\theta . 4}$

$= \frac{-1}{\sin 2\theta}$ (5)

$\left.\frac{dY}{dx}\right|_{\theta=\frac{\pi}{8}} = \frac{-1}{\sin\frac{\pi}{4}} = -\sqrt{2}$ (5)

The gradient of the normal is $\frac{1}{\sqrt{2}}$

The equation of normal.

$Y - 4\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\{x-(2+\cos\frac{\pi}{2})\}$

$Y - 2\sqrt{2} = \frac{1}{\sqrt{2}}(x-2)$

$x - \sqrt{2}Y + 2 = 0$ (5)

25

Q6] $\frac{d}{dx}\left\{\frac{1}{\sin x\cos x}\right\}$

$= (-1)(\sin x\cos x)^{-2}\{\sin x(-\sin x) + \cos x.\cos x\}$ (5)

$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x\cos^2 x}$

$= \frac{2\sin^2 x - 1}{\sin^2 x\cos^2 x} = \frac{2}{\cos^2 x} - \frac{1}{\sin^2 x\cos^2 x}$ (5)

$\int\left\{\frac{2}{\cos^2 x} - \frac{1}{\sin^2 x\cos^2 x}\right\}dx = \frac{1}{\sin x\cos x} + c$ (5)

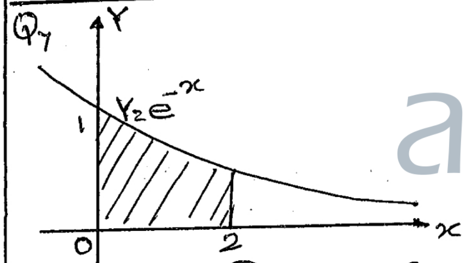
$2\tan x - \int \text{cosec}^2 x\sec^2 x\,dx = \frac{1}{\sin x\cos x} + c$ (5)

$\int \text{cosec}^2 x.\sec^2 x\,dx = 2\tan x - \frac{1}{\sin x\cos x} + c'$ (5)

; c' - arbitary constant

$\boxed{25}$

Q7



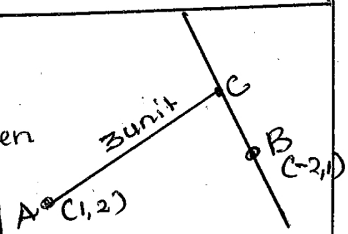
I) $A = \int_0^2 e^{-x}dx = \left.\frac{e^{-x}}{-1}\right|_0^2 = 1 - e^{-2}$ (5) (5)

II) $V = \pi\int_0^2 (e^{-x})^2 dx = \pi\left\{\frac{e^{-2x}}{-2}\right\}_0^2$ (5) (5)

$= -\frac{\pi}{2}(e^{-4}-1) = \frac{\pi}{2}(1-e^{-4})$ (5)

$\boxed{25}$

Q8]



The distance between A&C

$AC = \frac{|a\times1 + b\times2 + c|}{\sqrt{a^2+b^2}} = 3$ —(1) (5)

B is on the line $ax+by+c=0$

So $-2a+b+c=0$ (5)

$c = 2a-b$ —(2)

(1)&(2) $\frac{|a+2b+2a-b|}{\sqrt{a^2+b^2}} = 3$ (5)

$(3a+b)^2 = 9(a^2+b^2)$

$8b^2 = 6ab \Rightarrow b = \frac{3a}{4}$ $c = \frac{5a}{4}$ (5)

$\therefore ax + \frac{3a}{4}y + \frac{5a}{4} = 0$

$4x+3y+5=0$ (5)

$\boxed{25}$

Q9] S is a centre $O=(1,2)$ & radius 4 unit

$S \equiv (x-1)^2+(y-2)^2 = 4^2$ (5)

$x^2+y^2-2x-4y-11=0$

$S' \equiv x^2+y^2-10x-10y+c'=0$ (5)

[centre is (5,5)]

S & S' are Intersect orthogonally

$g=1$ $f=2$ $c=-11$ $g'=5$ $f'=5$

$2gg'+2ff'=c+c'$ (5)

$2\times1\times5+2\times2\times5 = -11+c'$

$c'=41$ (5)

$S' \equiv x^2+y^2-10x-10y+41=0$ (5)

$\boxed{25}$

Q10]

$2\tan^{-1}(1/3) + \cot^{-1}(3/4) = \pi/2$

$\tan^{-1}(1/3)=\alpha$ $\cot^{-1}(3/4)=\beta$

$0<\alpha,\beta<\pi/4$

$2\alpha+\beta = \pi/2$

$0<2\alpha = \pi/2-\beta < \pi/2$ (5)

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{\frac{2}{3}}{1-1/9} = \frac{2\times9}{3\times8}$ (5)

$= \frac{3}{4}$ —(1) (5)

$\tan(\frac{\pi}{2}-\beta) = \cot\beta = \frac{3}{4}$ —(2) (5)

(1)&(2) $\tan 2\alpha = \tan(\pi/2-\beta)$

$2\alpha = \pi/2-\beta$ (5)

$2\alpha+\beta = \pi/2$

(02)

Part B

11. (a) When $f(x)$ is divided by $(x-a)(x-b)$, the remainder is of the form $Ax+B$

i.e. $f(x) = (x-a)(x-b)\phi(x) + Ax + B$ (10)

$f(a) = Aa + B$ (5)

$f(b) = Ab + B$ (5)

$$A = \frac{f(a) - f(b)}{a - b} \quad (5)$$

$$B = \frac{af(b) - bf(a)}{a - b} \quad (5)$$

∴ remainder

$$= \frac{f(a) - f(b)}{a - b}x + \frac{af(b) - bf(a)}{a - b} \quad (5)$$

$\boxed{35}$

$f(x) = x^3 + \lambda x^2 + \mu x - 1$

$a = 2, b = -1$

$$\left.\begin{aligned} f(2) &= 4\lambda + 2\mu + 7 \\ f(-1) &= \lambda - \mu - 2 \end{aligned}\right\} (5)$$

$$\frac{f(a) - f(b)}{a - b} = \lambda + \mu + 3 \quad (5)$$

$$\frac{af(b) - bf(a)}{a - b} = 2\lambda + 1 \quad (5)$$

$(\lambda + \mu + 3)x + (2\lambda + 1) \equiv 5$ (5)

$$\left.\begin{aligned} \lambda + \mu + 3 &= 0 \\ 2\lambda + 1 &= 5 \end{aligned}\right\} (5)$$

$\lambda = 2$, (5) $\mu = -5$ (5)

$\boxed{35}$

(b)(i) $x^2 + ax + b^2 \equiv k(x - \alpha)(x - \beta)$

$x^2 + ax + b^2 \equiv k(x^2 - (\alpha + \beta)x + \alpha\beta)$

Equating the coefficients (10)

$$\left.\begin{aligned} x^2 // \quad 1 &= k \\ x // \quad a &= -k(\alpha + \beta) \\ x^0 // \quad b^2 &= k\alpha\beta \end{aligned}\right\} (10)$$

$\alpha + \beta = -a$, $\alpha\beta = b^2$ $\boxed{20}$

(ii) $x^2 + ax + b^2 = 0$

$\Delta = a^2 - 4(1)(b^2) = a^2 - 4b^2$ (10)

$|a| \geq 2|b| \Leftrightarrow$ (5) $a^2 - 4b^2 \geq 0$ (5)

$\Leftrightarrow$ roots are real. $\boxed{20}$

(iii) $|a| \geq 2|b| \Rightarrow \alpha$ and β are real

$(|\alpha| + |\beta|)^2 = \alpha^2 + \beta^2 + 2|\alpha\beta|$

$= (\alpha + \beta)^2 - 2\alpha\beta + 2|\alpha\beta|$ (5)

$= a^2 - 2b^2 + 2|b^2|$ (5)

$= a^2 - 2b^2 + 2b^2$

$= a^2$

$|\alpha| + |\beta| = |a|$ (5)

$|\alpha||\beta| = |\alpha\beta| = b^2$ (5)

Hence, the equation whose roots are $|\alpha|$ and $|\beta|$ is

$x^2 - |a|x + b^2 = 0$ (5) $\boxed{25}$

(iv) $|a| < 2|b| \Rightarrow \alpha$ and β are imaginary

$\Rightarrow \alpha$ and β are complex

$\therefore \beta = \bar{\alpha}$

$\alpha\beta = b^2 \Rightarrow \alpha\bar{\alpha} = b^2$ (5) $\Rightarrow |\alpha|^2 = b^2$

$\Rightarrow |\alpha| = |\beta| = |b|$ (5)

Hence, the equation whose roots are $|\alpha|$ and $|\beta|$ is

$x^2 - |b|x + b^2 = 0$ (5) $\boxed{15}$

$\boxed{103}$

12. (a)

(i) The commitee consist of 3 boys and 2 girls

No. of ways $= {}^6C_3 \cdot {}^5C_2$ (10)

$= 200.$ (5) $\boxed{15}$

(ii) Total No. of ways $= {}^{11}C_5$ (5)

The No. of ways that have no boys is ${}^5C_5 = 1$ (5) (5)

The no. of ways that have no girls is ${}^6C_5 = 6$ (5) (5)

$\therefore$ The required no. of ways is ${}^{11}C_5 - ({}^5C_5 + {}^6C_5)$ (5)

$= 462 - 1 - 6$

$= 455.$ (5) $\boxed{35}$

(iii) No. of ways the team can be selected if both particular boy and girl are in the team

$= {}^9C_3 = 84$ (10) (5)

$\therefore$ The required no. of ways

$= 455 - 84$ (5)

$= 371$ (5) $\boxed{25}$

(b) $U_r = f(r) - f(r+1)$

$$\frac{r}{(r+1)(r+2)(r+3)} = \frac{\lambda r + \mu}{(r+1)(r+2)} - \frac{\lambda(r+1)+\mu}{(r+2)(r+3)}$$

$r \equiv (\lambda r + \mu)(r+3) - (\lambda r + \lambda + \mu)(r+1)$ (10)

$r \equiv \lambda r + 2\mu - \lambda$

$\lambda = 1, \quad 2\mu - \lambda = 0$ (5)

$\lambda = 1, \quad \mu = \frac{1}{2}$ (5) (5) $\boxed{25}$

$U_r = f(r) - f(r+1)$

$r=1 \quad U_1 = f(1) - \cancel{f(2)}$ (5)

$r=2 \quad U_2 = \cancel{f(2)} - \cancel{f(3)}$

$\vdots$

$r=n-1 \quad U_{n-1} = \cancel{f(n-1)} - \cancel{f(n)}$ (5)

$r=n \quad U_n = \cancel{f(n)} - f(n+1)$

$$\sum_{r=1}^{n} U_r = f(1) - f(n+1) \quad (5)$$

$$\Rightarrow \sum_{r=1}^{n} U_r = \frac{1}{4} - \frac{2n+3}{2(n+2)(n+3)} \quad (5) \quad \boxed{20}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \lim_{n\to\infty} \left\{ \frac{1}{4} - \frac{\frac{2}{n} + \frac{3}{n^2}}{2(1+\frac{2}{n})(1+\frac{3}{n})} \right\} \quad (5)$$

$= \frac{1}{4} - 0$

$= \frac{1}{4}$ (5)

$\therefore \sum_{r=1}^{\infty} U_r$ converges (5)

$$\sum_{r=3}^{\infty} U_r = \sum_{r=1}^{\infty} U_r - U_1 - U_2 \quad (5)$$

$= \frac{1}{4} - \frac{1}{24} - \frac{1}{30}$ (5)

$= \frac{7}{40}.$ (5) $\boxed{30}$

13. (a) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ $\boxed{5}$

$z = \cos\theta + i\sin\theta$

$z^n = \cos n\theta + i\sin n\theta$ (5)

$$\frac{1}{z^n} = \frac{1}{\cos n\theta + i\sin n\theta}$$

$$= \frac{\cos n\theta - i\sin n\theta}{\cos^2 n\theta - i^2\sin^2 n\theta} \quad (5)$$

$= \cos n\theta - i\sin n\theta$ (5)

$z^n + \frac{1}{z^n} = 2\cos n\theta$ (5)

$z^n - \frac{1}{z^n} = 2i\sin n\theta$ (5) $\boxed{25}$

$\boxed{104}$

$$\frac{z^{2n}-1}{z^{2n}+1}$$

$$= \frac{z^n - \frac{1}{z^n}}{z^n + \frac{1}{z^n}} \quad 5$$

$$= \frac{2i\sin n\theta}{2\cos n\theta} \quad 5 \qquad \boxed{10}$$

$$= i\tan n\theta$$

(ii) $\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6$

$$= \left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)^6 + \left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)^6$$

$$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^6 + \left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)^6 \quad 10$$

$$= \cos\frac{6\pi}{6} + i\sin\frac{6\pi}{6} + \left(\cos\frac{30\pi}{6} + i\sin\frac{30\pi}{6}\right) \quad 5$$

$$= \cos\pi + i\sin\pi + \cos 5\pi + i\sin 5\pi$$

$$= -1 + 0 - 1 + 0$$

$$= -2 \quad 5 \qquad \boxed{20}$$

(b) $z_1 = \frac{1+i}{1-i}$

$$= \frac{(1+i)^2}{1-i^2} \quad 5$$

$$= \frac{1+2i+i^2}{2} \quad 5$$

$$= \frac{1+2i-1}{2}$$

$$= i$$

$$= 0 + i(1) \quad 5$$

$z_2 = \frac{\sqrt{2}}{1-i}$

$$= \frac{\sqrt{2}(1+i)}{1-i^2} \quad 5 \qquad \boxed{25}$$

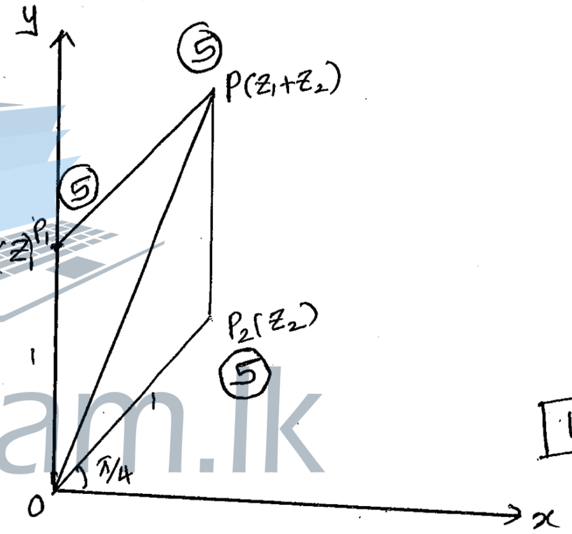
$$= \frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}}\right) \quad 5$$

$z_1 = 0 + i(1) = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$|z_1| = 1$ 5, $\mathrm{Arg}(z_1) = \frac{\pi}{2}$ 5

$z_2 = \frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}}\right) = 1\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$|z_2| = 1$ 5, $\mathrm{Arg}(z_2) = \frac{\pi}{4}$ 5 $\qquad \boxed{20}$



$\boxed{15}$

In the parallelogram OP_2PP_1,

$OP_1 = OP_2$

$\therefore OP_2PP_1$ is a rhombus 5

$P_2\hat{O}P = P\hat{O}P_1 = \frac{\pi}{8}$ 5

$x\hat{O}P = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$ 5

$$z_1 + z_2 = \frac{1+i}{1-i} + \frac{\sqrt{2}}{1-i}$$

$$= \frac{1+\sqrt{2}+i}{1-i} \quad 5$$

$\mathrm{Arg}\left(\frac{1+\sqrt{2}+i}{1-i}\right) = \mathrm{Arg}(z_1+z_2) = \frac{3\pi}{8}$ 5

Also $z_1 + z_2 = \frac{1}{\sqrt{2}} + i\left(1 + \frac{1}{\sqrt{2}}\right)$

$$\tan\frac{3\pi}{8} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}+1 \quad 5 \qquad \boxed{30}$$

105

14. (a)

$$f(x) = \frac{1-x}{x^2}$$

$$f'(x) = \frac{x^2(-1) - (1-x)2x}{x^4} \quad (10)$$

$$= \frac{x-2}{x^3} \quad (5)$$

$$f''(x) = \frac{x^3(1) - (x-2)3x^2}{x^6} \quad (10)$$

$$= \frac{-2x^3 + 6x^2}{x^6} \quad (5)$$

$$= -\frac{2(x-3)}{x^4} \quad \boxed{30}$$

When $y = 0$, $x = 1$

$$\lim_{x \to 0} \frac{1-x}{x^2} = \infty$$

vertical asymptete : $x = 0$ (5)

$$\lim_{x \to \pm\infty} \frac{1-x}{x^2} = \lim_{x \to \pm\infty} \left(\frac{1}{x^2} - \frac{1}{x}\right) = 0$$

Horizontal asymptote ; $y = 0$ (5)

When $f'(x) = 0 \Rightarrow x = 2$ (5)

$f(2) = -\frac{1}{4}$

$x < 0$	$0 < x < 2$	$x > 2$
$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
Increasing	decreacing	increasing
(5)	(5)	(5)

$(2, -\frac{1}{4})$ is a local minmum

(5)

$\boxed{106}$

when

$f''(x) = 0$, (5)

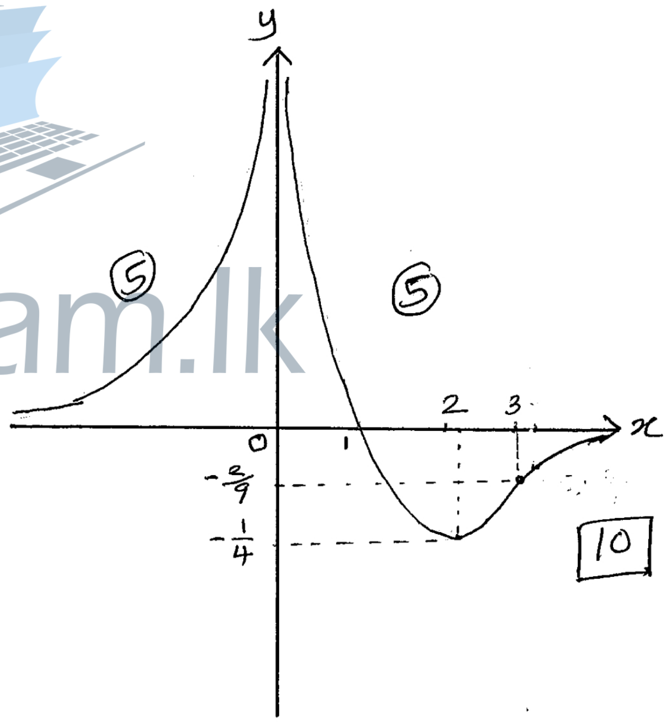
When $f''(x) = 0 \Rightarrow x = 3$

$f(3) = -\frac{2}{9}$

$x < 0$	$0 < x < 3$	$x > 3$
$f''(x) > 0$	$f''(x) > 0$	$f''(x) < 0$
concave up	concave up	concave down
(5)	(5)	(5)

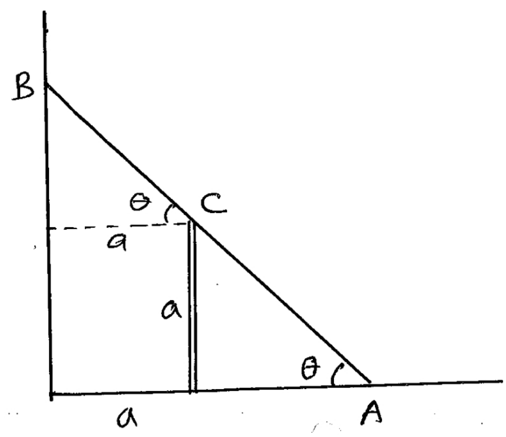
$(3, -\frac{2}{9})$ is a point of inflection $\boxed{60}$

(5)



$\boxed{10}$

(b)



(i)

$AB = AC + CB$

$y = a\,\text{cosec}\theta + a\sec\theta$ (5)

(ii) $\frac{dy}{d\theta} = a(-\text{cosec}\theta\cot\theta) + a\sec\theta\tan\theta$ (10)

$\frac{dy}{d\theta} = a\,\text{cosec}\theta\cot\theta\,(-1+\tan^3\theta)$

$\frac{dy}{d\theta} = a\,\text{cosec}\theta\cot\theta\,(\tan^3\theta - 1)$ (5)

$\frac{dy}{d\theta} = 0 \Rightarrow \tan\theta = 1$ (5)

$\theta = \frac{\pi}{4}$ (5)

$0<\theta<\pi/4$	$\theta=\frac{\pi}{4}$	$\frac{\pi}{4}<\theta<\pi/2$
$\frac{dy}{d\theta}<1$	$\frac{dy}{d\theta}=0$	$\frac{dy}{d\theta}>1$

(10)

$\therefore$ y is minimum when $\theta = \frac{\pi}{4}$ (5)

$y_{min} = \sqrt{2}a + \sqrt{2}a$

$= 2\sqrt{2}a$ m (5)

$\boxed{50}$

15.

a) $\frac{1}{x^3-8} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$

$1 \equiv A(x^2+2x+4) + (Bx+C)(x-2)$

$x=2$; $1 = 12A$

$A = \frac{1}{12}$

x^2; $0 = A+B$

$B = -\frac{1}{12}$

x^0; $1 = 4A - 2C$

$C = -\frac{1}{3}$

$A = \frac{1}{12}$, $B = -\frac{1}{12}$, $C = -\frac{1}{3}$ (15)

$\frac{1}{x^3-8} = \frac{\frac{1}{12}}{x-2} + \frac{-\frac{1}{12}x - \frac{1}{3}}{x^2+2x+4}$

$= \frac{1}{12(x-2)} - \frac{(x+4)}{12(x^2+2x+4)}$

$\int \frac{1}{x^3-8}dx = \frac{1}{12}\int\frac{1}{x-2}dx - \frac{1}{12\times 2}\int\frac{2(x+4)}{x^2+2x+4}dx$ (5)

$= \frac{1}{12}\int\frac{1}{x-2}dx - \frac{1}{24}\int\frac{(2x+2)}{x^2+2x+4}dx$ (5)

$- \frac{1}{24}\int\frac{6}{(x+1)^2+(\sqrt{3})^2}dx$ (5)

$= \frac{1}{12}\ln|x-2|$ (5) $- \frac{1}{24}\ln(x^2+2x+4)$ (5)

$- \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$ (5) $+k$ (5)

where k is an arbitary constant

$\boxed{50}$

07

b)

(i) $\int x^n (\ln x)^2 dx$

$= (\ln x)^2 \frac{x^{n+1}}{(n+1)} - \int \frac{x^{n+1}}{(n+1)} \cdot 2(\ln x)\frac{1}{x} dx$ (10)

$= \frac{x^{n+1}}{(n+1)}(\ln x)^2 - \frac{2}{(n+1)} \int x^n (\ln x) dx$ (5)

$= \frac{x^{n+1}}{(n+1)}(\ln x)^2 - \frac{2}{n+1}\left\{(\ln x)\frac{x^{n+1}}{(n+1)} - \int \frac{x^{n+1}}{(n+1)} \cdot \frac{1}{x} dx\right\}$ (10)

$= \frac{x^{n+1}}{(n+1)}(\ln x)^2 - \frac{2x^{n+1}}{(n+1)^2}(\ln x) + \frac{2}{(n+1)^2}\int x^n dx$ (5)

$= \frac{x^{n+1}}{(n+1)}(\ln x)^2 - \frac{2x^{n+1}}{(n+1)^2}(\ln x) + \frac{2x^{n+1}}{(n+1)^3}$ (5) $+ C$

Where C is an arbitary Constant $\boxed{35}$

(ii) $\int_1^2 \frac{(\ln x)^2}{x} dx$

$= \left\{\frac{(\ln x)^3}{3}\right\}_1^2$ (5)

$= \frac{1}{3}\left\{(\ln 2)^3 - (\ln 1)^3\right\}$

$= \frac{1}{3}(\ln 2)^3$ (5)

$\boxed{10}$

c) $t = \tan\frac{x}{2}$

$\frac{dt}{dx} = \sec^2\frac{x}{2} \cdot \frac{1}{2}$

$= \frac{(1+t^2)}{2}$ (5)

$x = 0 \Rightarrow t = 0$
$x = \pi \Rightarrow t = \infty$ (5)

$\sin x = \frac{2t}{1+t^2}$

$\int_0^{\pi} \frac{1}{1+\sin x} dx$

$= \int_0^{\infty} \frac{1}{1+\left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{(1+t^2)}$ (5)

$= \int_0^{\infty} \frac{2}{(t+1)^2} dt$

$= 2\left\{\frac{(t+1)^{-1}}{-1}\right\}_0^{\infty}$ (5)

$= -2\left\{\frac{1}{\infty} - \frac{1}{1}\right\} = 2$ (5) $\boxed{25}$

$\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)} dx$ (5)

$= \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\sin x} dx$ (5)

$\Rightarrow 2\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx = \pi\int_0^{\pi} \frac{\sin x}{1+\sin x} dx$ (5)

$= \pi\int_0^{\pi} \frac{(1+\sin x) - 1}{(1+\sin x)} dx$ (5)

$= \pi\int_0^{\pi} 1 \cdot dx - \pi\int_0^{\pi} \frac{1}{1+\sin x} dx$

$= \pi[x]_0^{\pi} - \pi \times 2$ (5)

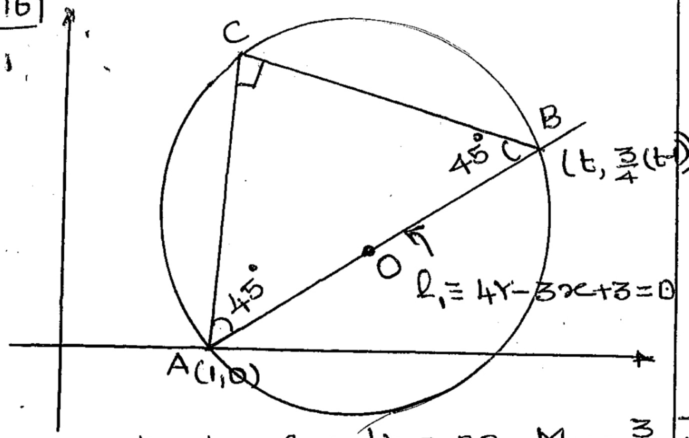
$= \pi^2 - 2\pi$

$\Rightarrow \int_0^{\pi} \frac{x\sin x}{1+\sin x} dx = \frac{\pi}{2}(\pi - 2)$ (5)

$\boxed{30}$

$\boxed{08}$

Q16)



The gradient of a line $AB = M_{AB} = \frac{3}{4}$

The gradient of a line $AC = M_{AC} = m_1$

$$\tan 45^\circ = \left|\frac{\frac{3}{4} - m_1}{1 + \frac{3}{4}m_1}\right| = \left|\frac{3-4m_1}{4+3m_1}\right| = 1 \quad (5)$$

$$\Rightarrow \frac{3-4m_1}{4+3m_1} = \pm 1 \quad (+)\ 3-4m_1 = 4+3m_1$$

$$m_1 = -\frac{1}{7} \quad (5)$$

or $(-)\ 3-4m_1 = -4-3m_1$

$$m_1 = 7 \quad (5)$$

ΔABC on the 1^{st} Quadrant (5)

So $M_{AC} > 0$ $\quad m_1 = 7$

equation of line AC

$$l_3 \equiv \frac{Y-0}{x-1} = 7 \quad (5)$$

$$l_3 = Y - 7x + 7 = 0 \quad (5) \quad \boxed{30}$$

let $B \equiv (t, \frac{3}{4}(t-1))$ (5)

From the Phythagorous theorem

$$(t-1)^2 + \frac{3^2}{4^2}(t-1)^2 = 25 \quad (5)$$

$$25(t-1)^2 = 25 \times 16 \Rightarrow t - 1 = \pm 4 \quad (5)$$

(+)ve $t = 5$ $\quad$ (−)ve $t = -3$ (5)

ΔABC on the 1^{st} Quadrant

So $t = 5$ (5)

The Pair of $B \equiv (5, 3)$ (5) $\boxed{30}$

The gradient of line $BC = M_{BC} = m_2$

$AC \perp BC$ (5)

$$m_2 = -\frac{1}{7} \quad (5)$$

equation of line BC

$$l_2 \equiv \frac{Y-3}{x-5} = \frac{-1}{7} \quad (5)$$

$$l_2 \equiv 7Y + x - 26 = 0 \quad (5)$$

The Area of $\Delta ABC = \frac{1}{2} \times AC \times BC$

$$= \frac{1}{2} \times \frac{5}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \quad (5)$$

$$(5) = \frac{25}{4} \text{ Square unit} \quad \boxed{30}$$

b) The equation of the circle S'

$$S' \equiv \frac{(Y-0)}{(x-1)} \cdot \frac{(Y-3)}{(x-5)} = -1 \quad (10)$$

$$S' \equiv Y^2 - 3Y = -x^2 + 6x - 5$$

$$S' \equiv x^2 + Y^2 - 6x - 3Y + 5 = 0 \quad (5)$$

O is the centre of the circle $S' = 0$

$O \equiv (3, \frac{3}{2})$ (5) & radious $r_1 = \frac{5}{2}$ unit

The circle $S'' = 0$ is through the Points B & C.

So General equation.

$$S'' \equiv S' + \lambda l_2 = 0$$

$$S'' \equiv (x^2 + Y^2 - 6x - 3Y + 5) + \lambda(7Y + x - 26) = 0 \quad (10)$$

$S'' = 0$ is through the centre of Circle $S' = 0$

$$\text{So } (9 + \frac{9}{4} - 18 - \frac{9}{2} + 5) + \lambda(\frac{21}{2} + 3 - 26) = 0 \quad (5)$$

$$\frac{25\lambda}{2} = \frac{25}{4} \Rightarrow \lambda = \frac{1}{2} \quad (5)$$

$$S'' \equiv x^2 + Y^2 - \frac{11x}{2} + \frac{Y}{2} - 8 = 0 \quad (5)$$

O' is the centre of the circle $S'' = 0$

$O' \equiv (\frac{11}{4}, -\frac{1}{4})$ (5)

The distance between two centre Points O & O'

$$OO'^2 = (3 - \frac{11}{4})^2 + (\frac{3}{2} + \frac{1}{4})^2 = \frac{50}{16} \quad (5)$$

$$OO' = \frac{5}{2\sqrt{2}} < \frac{5}{2} = r_1 \quad (5)$$

So O' is inside the circle $S' = 0$

$\boxed{60}$

$\boxed{09}$

Q17 $\tan 4\theta - \cot 2\theta = 0 \quad (-\frac{\pi}{2} < \theta < \frac{\pi}{2})$

$\tan 4\theta = \cot 2\theta = \tan(\frac{\pi}{2} - 2\theta)$ (5)

$4\theta = n\pi + \frac{\pi}{2} - 2\theta$ (5)

$6\theta = \frac{(2n+1)\pi}{2}$

$\theta = \frac{(2n+1)\pi}{12}$ (5); $n \in \mathbb{Z}$

$\theta = \pm\frac{5\pi}{12}, \pm\frac{3\pi}{12}, \pm\frac{\pi}{12}$ (10) $\boxed{25}$

---

ii) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ (5)

If $A = B = \theta$

$\tan(\theta+\theta) = \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta \cdot \tan\theta}$ (5)

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ (5) $\boxed{15}$

---

iii $\tan 4\theta = \dfrac{2\tan 2\theta}{1-\tan^2 2\theta}$ (5)

$= \dfrac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]^2}$ (5)

$= \dfrac{4\tan\theta[1-\tan^2\theta]}{(1-\tan^2\theta)^2 - 4\tan^2\theta}$ (5)

$= \dfrac{4\tan\theta[1-\tan^2\theta]}{\tan^4\theta - 6\tan^2\theta + 1}$

$\tan 4\theta - \cot 2\theta = \tan 4\theta - \dfrac{1}{\tan 2\theta}$ (5)

$= \dfrac{4\tan\theta(1-\tan^2\theta)}{\tan^4\theta - 6\tan^2\theta+1}$ (5) $- \dfrac{(1-\tan^2\theta)}{2\tan\theta}$ (5)

$= \dfrac{(1-\tan^2\theta)(4\tan^2\theta - \tan^4\theta - 1)}{2\tan\theta(\tan^4\theta - 6\tan^2\theta+1)}$

$t = \tan\theta$ (5)

$= \dfrac{(t^2-1)(t^4-14t^2+1)}{2t(t^4-6t^2+1)}$ (5)

$\boxed{40}$

$\tan 4\theta - \cot 2\theta = \dfrac{(t^2-1)(t^4-14t^2+1)}{2t(t^4-6t^2+1)} = 0$

$\tan 4\theta - \cot 2\theta = 0$ — (1)

$(t^2-1)(t^4-14t^2+1) = 0$ — (2)

The roots of equation (1) & roots of eq (2) both are same. (5)

$\tan(\pm\frac{\pi}{4}) = \pm 1$ are two roots of eq^n $t^2-1=0$ (5)

So $\tan(\pm\frac{\pi}{12})$, $\tan(\pm\frac{5\pi}{12})$ are four roots of $t^4-14t^2+1=0$ (5)

$(t^2-7)^2 - 48 = 0 \Rightarrow t^2 = 7 \pm 4\sqrt{3}$ (5)

$t = \pm\sqrt{7+4\sqrt{3}}$ or $t = \pm\sqrt{7-4\sqrt{3}}$ (5) $(7 > 4\sqrt{3})$

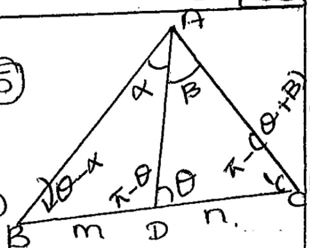
Therefore four roots are

$t = \pm\sqrt{7+4\sqrt{3}}$ or $t = \pm\sqrt{7-4\sqrt{3}}$

but $0 < \tan\frac{\pi}{12} < \tan\frac{5\pi}{12}$ (5)

$\tan\frac{\pi}{12} = 7-4\sqrt{3}$ & $\tan\frac{5\pi}{12} = 7+4\sqrt{3}$ (5) $\boxed{35}$

$\boxed{10}$



$\dfrac{m}{n} = \dfrac{BD}{DC} = \dfrac{BD}{AD} \cdot \dfrac{AD}{DC}$ (5)

$\dfrac{m}{n} = \dfrac{\sin\alpha}{\sin(\theta-\alpha)}$ (5) $\cdot \dfrac{\sin(\theta+\beta)}{\sin\beta}$ (5)

$m\sin\beta\sin(\theta-\alpha) = n\sin\alpha\sin(\theta+\beta)$

$m\sin\beta(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ (5)

$= n\sin\alpha(\sin\theta\cos\beta + \sin\beta\cos\theta)$ (5)

$\sin\theta[m\sin\beta\cos\alpha - n\sin\alpha\cos\beta]$ (5)

$= (m+n)\sin\alpha\sin\beta\cos\theta$

both side divied $\sin\alpha\sin\beta\sin\theta$

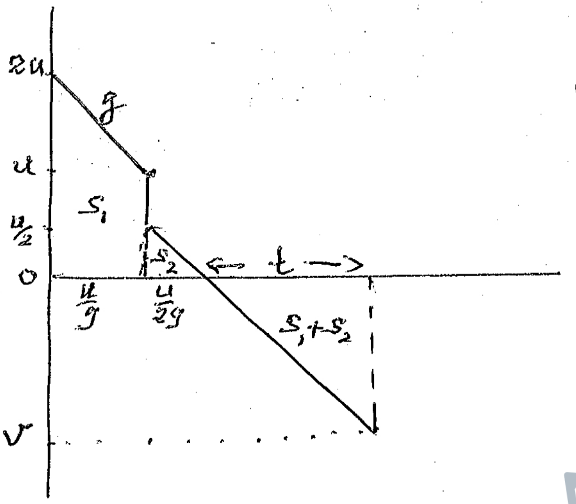
(5)

$m\cot\alpha - n\cot\beta = (m+n)\cot\theta$

$\boxed{35}$

G.C.E A/L Examination March - 2019
Fied Work Centre
FWC
Grade - 13 (2019) Combined Mathematics - II Marking Scheme

(1)



(10)

$S_1 = \frac{1}{2}(2u+u)\frac{u}{g} = \frac{3u^2}{2g}$

$S_2 = \frac{1}{2}\cdot\frac{u}{2}\cdot\frac{u}{2g} = \frac{u^2}{8g}$

Max. Height $= S_1+S_2 = \frac{13u^2}{8g}$ (5)

$\frac{1}{2}v.t = \frac{13u^2}{8g}$

$\frac{1}{2}\cdot\frac{v^2}{g} = \frac{13u^2}{8g}$

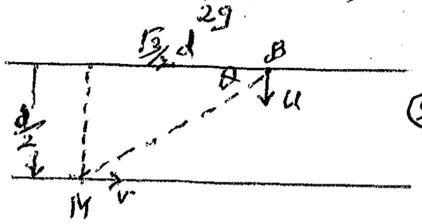
$v = \frac{\sqrt{13}u}{2}$

$t = \frac{\sqrt{13}}{2}\frac{u}{g}$ (5)

Total time $= \frac{u}{g}+\frac{u}{2g}+\frac{\sqrt{13}}{2}\frac{u}{g}$

$= \frac{u}{2g}(3+\sqrt{13})$. (5)

(2)

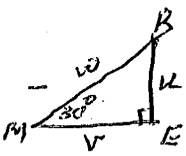


(5)

$\theta = 30°$ (5)

$V_{ME} \xrightarrow{} v$ $V_{BE} \downarrow u$ V_{BM} (5)

$V_{BE} = V_{BM} + V_{ME}$

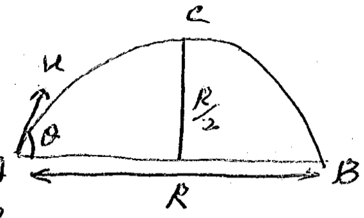


(5)

$u = \frac{v}{\sqrt{3}}$

$w = \frac{2v}{\sqrt{3}}$ (5)

(3) $A \to C$



$\uparrow v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2g.\frac{R}{2}$

$R = \frac{u^2\sin^2\theta}{g}$ (5)

$\uparrow A \to B$ $s = ut + \frac{1}{2}at^2$

$0 = u\sin\theta.t - \frac{1}{2}gt^2$

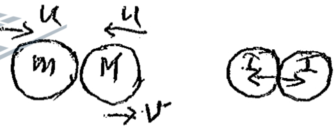
$t = \frac{2u\sin\theta}{g}$ (5)

$\to R = u\cos\theta.\frac{2u\sin\theta}{g}$

$= \frac{2u^2}{g}\sin\theta\cos\theta$ (5)

(5) $\tan\theta = 2 \Rightarrow R = \frac{u^2}{g}\cdot\left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4u^2}{5g}$ (5)

(4)



$m \to -I = m.0 - mu$ (5)

$M \to I = Mv - (-Mu)$ (5)

$0 = Mv - (m-M)u$

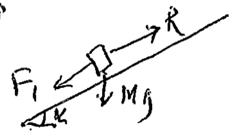
$v = \left(\frac{m-M}{M}\right)u$ (5)

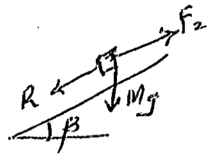
N.L.R $v = e.2u$ (5)

$\therefore 2eu = \left(\frac{m-M}{M}\right)u$

$m : M = (2e+1) : 1$ (5)

(5)





(5)

$\swarrow F_1 - R + Mg\sin\alpha = 0$ (5)

$F_1 = R - Mg\sin\alpha$

$H = F_1.2u$ (5)

$\nearrow F_2 - R - Mg\sin\beta = 0$

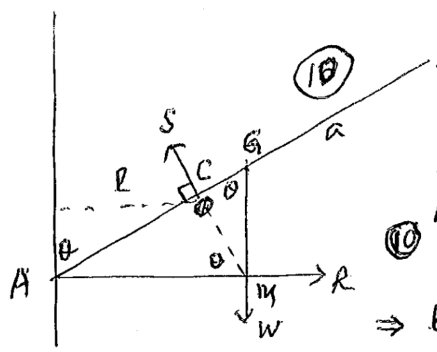
$F_2 = R + Mg\sin\beta$

$H = F_2.u$ (5)

$\Rightarrow R = Mg\left(\frac{1}{q}+\frac{2}{p}\right)$

$H = 2Mgu\left(\frac{1}{p}+\frac{1}{q}\right)$ (5)

(6)



$AM = R\sin\theta$

(10) $AC = AM\sin\theta$

$l = AC\sin\theta$

$\Rightarrow l = a\sin^3\theta$

$\sin\theta = \left(\frac{l}{a}\right)^{\frac{1}{3}}$ (5)

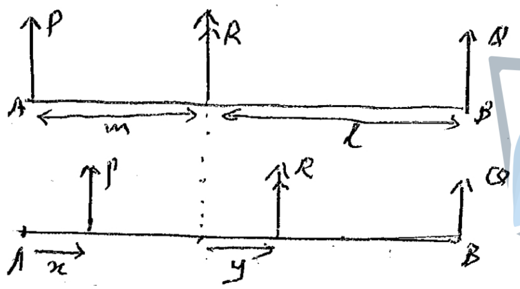
(7) $(\underline{a}-\underline{b}).(\underline{a}-\underline{b}) = |\underline{a}-\underline{b}|^2$ (10)

$a^2+b^2-2\underline{a}.\underline{b} = |\underline{a}-\underline{b}|^2$ (5)

$2^2+3^2-2.4 = |\underline{a}-\underline{b}|^2$ (5)

$|\underline{a}-\underline{b}| = \sqrt{5}$ (5)

(8)



(1) $\frac{P}{Q} = \frac{l}{m}$ (5)

(2) $\frac{P}{Q} = \frac{l-y}{m+y-x}$ (5)

$\Rightarrow \frac{P}{Q} = \frac{l}{m} = \frac{l-y}{m+y-x} = \frac{y}{x-y}$

$\Rightarrow P(x-y) = Qy$

$y = \frac{Px}{P+Q}$ (5)

(9) A, B, C. independant events

$\Rightarrow P(A\cap B) = P(A).P(B)$

$P(B\cap C) = P(B).P(C)$

$P(C\cap A) = P(C).P(A)$ (10)

$P(A\cap B\cap C) = P(A).P(B).P(C)$

$P(A'\cap B'\cap C') = P(A\cup B\cup C)'$

$= 1 - P(A\cup B\cup C)$ (5)

$= 1 - [P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)]$

$= 1 - [P(A)+P(B)+P(C)-P(A).P(B)-P(B).P(C)-P(C).P(A)+P(A).P(B).P(C)]$ (5)

$= 1-P(A)-P(B)(1-PA)-P(C)(1-PB)-P(C)P(A)(1-PB)$

$= [1-P(A)][1-P(B)]-(1-PB)P(C)(1-PA)$

$= P(A').P(B')-P(B')P(C)(P(A'))$

$= P(A').P(B')(1-P(C))$

$= P(A')P(B')P(C')$ (5)

$\Rightarrow$ A', B', C' Independants.

(10) $P(A\cup B) = 1$ (5), $P(A) = \frac{1}{4}$

$\frac{P(A\cap B)}{P(A)} = \frac{2}{5}$ (5)

$P(A\cap B) = \frac{2}{5}.\frac{1}{4} = \frac{1}{10}$ (5)

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$ (5)

$1 = \frac{1}{4}+P(B)-\frac{1}{10}$

$P(B) = \frac{17}{20}$ (5)

(11) For A $v^2 = u^2+2as$

$v^2 = 4u^2-2g.\frac{3u^2}{2g}$

$v = u$ (10)

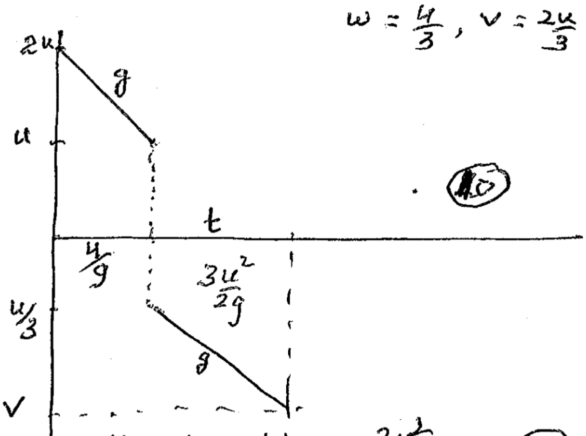
A $\downarrow I = 2mw+2mu$ (10)

B $\uparrow I = mv+2mu$

$v = 2w$

NLR $v+w = \frac{1}{3}\times 3u = u$

$w = \frac{u}{3}$, $v = \frac{2u}{3}$ (10)



$\frac{1}{2}\left(\frac{u}{3}+\frac{u}{3}+gt\right) = \frac{3u^2}{2g}$ (10)

$3g^2t^2+2ugt-9u^2 = 0$

$t = \frac{u}{3g}(2\sqrt{7}-1)$ (10)

Total time $= \frac{u}{g}+t$

$= \frac{2u}{3g}(\sqrt{7}+1)$ (10)

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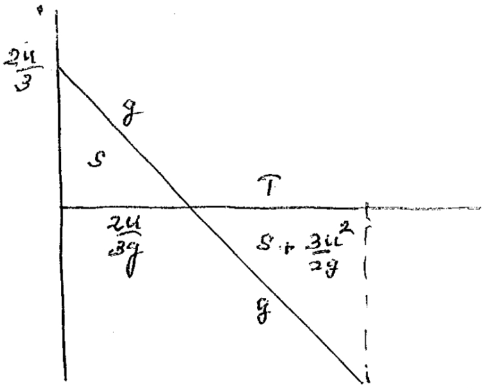


(5)

$s = \frac{2u^2}{9g}$ $\qquad \frac{1}{2}gT^2 = \frac{2u^2}{9g} + \frac{3u^2}{2g}$

$T = \frac{u\sqrt{31}}{3g}$ (5)

Total time $= \frac{2u}{3g} + \frac{u\sqrt{31}}{3g}$

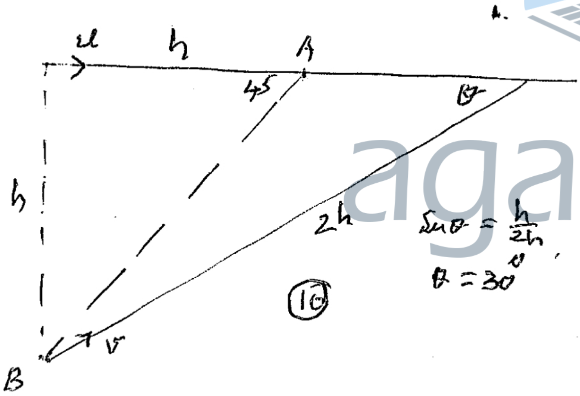
$= \frac{u}{3g}(2+\sqrt{31})$

11 b) I ↓ $s = ut + \frac{1}{2}at^2$

$h = \frac{1}{2}gt^2$ (10) $\qquad u = \sqrt{\frac{gh}{2}}$

$t = \sqrt{\frac{2h}{g}}$

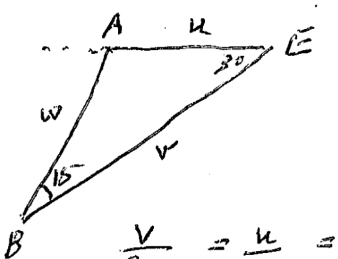
II Distance covered by A $= \sqrt{\frac{gh}{2}} \cdot \sqrt{\frac{2h}{g}} = h$ (10)



$\sin\theta = \frac{h}{2h}$

$\theta = 30^0$ (10) (10)

$V_{BA} = V_{BE} + V_{EA}$



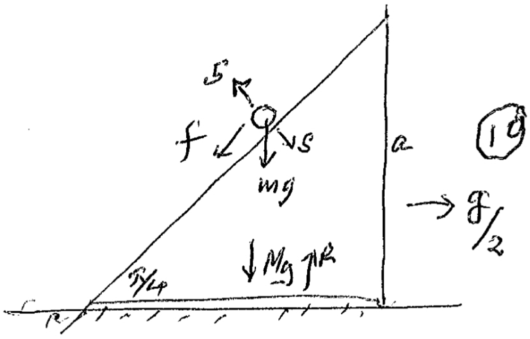
(10)

$\frac{v}{\sin 45} = \frac{u}{\sin 15} = \frac{w}{\sin 30}$

$v = \frac{\sqrt{2}}{2}u \operatorname{cosec} 15$

$w = \frac{1}{2}u \operatorname{cosec} 15$ (10)

(12) a)



(10)

a_{PW} f ↙ $\qquad a_{WE} \to \frac{g}{2}$ (10)

For system $\to$ $0 = M\frac{g}{2} + m(\frac{g}{2} - f\cos 45)$ (15)

$\sqrt{2}mf = (M+m)g$ ——— (1)

m ↙ $mg\sin\frac{\pi}{4} = m(f - \frac{g}{2}\cos 45)$ (15)

$f = \frac{3g}{2\sqrt{2}}$ ——— (2)

(1)+(2) ⇒ $\sqrt{2}m \cdot \frac{3g}{2\sqrt{2}} = (M+m)g$ (5)

$m = 2M$

m ↙ $s = ut + \frac{1}{2}at^2$

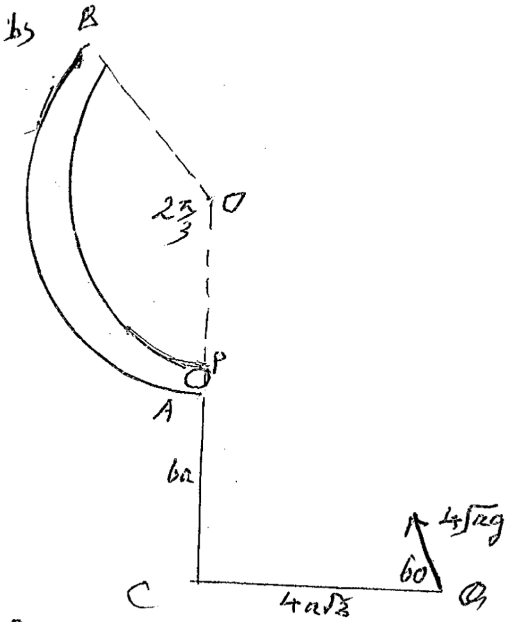
$\sqrt{2}a = \frac{1}{2} \cdot \frac{3}{2\sqrt{2}} g t^2$

$t = \sqrt{\frac{8a}{3g}}$ (5)

$M \to$ $s = \frac{1}{2} \cdot \frac{g}{2} \cdot \frac{8a}{3g}$

$= \frac{2}{3}a$ (5)

b)



for Q ← $4a\sqrt{3} = 4\sqrt{ag}\cos 60 \cdot t$ (10)

$t = 2\sqrt{\frac{3a}{g}}$

↑ $s = 4\sqrt{ag}\sin 60 \cdot 2\sqrt{\frac{3a}{g}} - \frac{1}{2}g \cdot 4 \cdot \frac{3a}{g}$

$= 6a$ (10)

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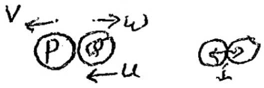
↑ $v = u + at$

$v = 4\sqrt{ag}\sin 60 - g \cdot 2\sqrt{\frac{3a}{g}}$

$= 0$ (10)

So, Q, directly impact P at A horizontally (5)



$u = 2\sqrt{ag}$

For P $\leftarrow I = mV - m.0$ (10)

Q $\rightarrow I = mw + mu$

$V = w + u$

NLR $V + w = 1.u$

$V - w = u$ (10)

$V = u, \; w = 0$ (5)

P moves with velocity $u = 2\sqrt{ag}$

Conservation of energy

$\frac{1}{2}m\,4ag - mga = mg\frac{a}{2} + \frac{1}{2}mV^2$ (10)

$V = \sqrt{ag}$ (5)

iv ↑ $v^2 = u^2 + 2as$

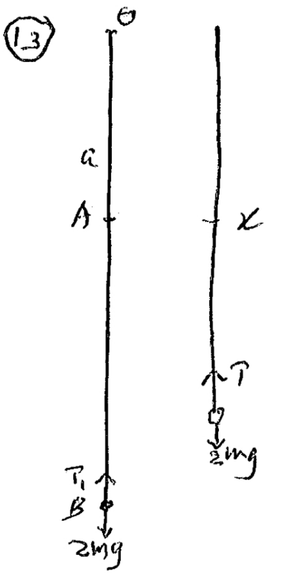
$0 = ag\sin^2 60 - 2gH$

$H = \frac{3a}{8}$ (5)

Maximum height above C $= 6a + a + \frac{a}{2} + \frac{3a}{8}$

$= \frac{63}{8}a$ (10)

(13)



$T = \frac{\lambda}{a}\cdot 2a = 2mg$

$\lambda = mg$ (10)

$I = \Delta mv$

$4m\sqrt{ag} = 2mV$

$V = 2\sqrt{ag}$ (10)

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$2mg - T = 2m\ddot{x}$ (10)

$2mg - \frac{mg}{a}(x - a) = 2m\ddot{x}$

$\ddot{x} + \frac{g}{2a}(x - 3a) = 0$ (10)

$X = x - 3a \Rightarrow \ddot{X} = \ddot{x}$

$\therefore \ddot{X} = -\frac{g}{2a}X$ (10)

(i) SHM with centre $X = 0 \Rightarrow x = 3a$ (10)

$x - 3a = \alpha\cos\omega t + \beta\sin\omega t$

$\dot{x} = -\alpha\omega\sin\omega t + \beta\omega\cos\omega t$

$\ddot{x} = -\alpha\omega^2\cos\omega t - \beta\omega^2\sin\omega t$

$\ddot{x} = -\omega^2(x - 3a)$ (10)

$\omega = \sqrt{\frac{g}{2a}}$

$x = 3a, \; t = 0, \; \dot{x} = 2\sqrt{ag}$

$\alpha = 0$ (5)

$2\sqrt{ag} = \beta\omega, \; \beta = 2\sqrt{2}a$ (5)

$x - 3a = 2\sqrt{2}a\sin\omega t$

$\dot{x} = 2\sqrt{2}a\omega\cos\omega t$ (5)

$\ddot{x} = -2\sqrt{2}a\omega^2\sin\omega t$ (5)

$\ddot{x} = -\omega^2(x - 3a)$ (10)

$\dot{x} = 0 \Rightarrow x = 3a + 2\sqrt{2}a$ (10)

centre B, amplitude $2\sqrt{2}a$ (10)

$x = a \Rightarrow$ Tension become zero.

$x = a \Rightarrow a - 3a = 2\sqrt{2}a\sin\omega t$.

$\sin\omega t_1 = -\frac{1}{\sqrt{2}}$

$\omega t_1 = \frac{5\pi}{4}$

$t_1 = \frac{5\pi}{4}\sqrt{\frac{2a}{g}}$ (10)

$t = t_1, \; \dot{x} = v$.

$v = 2\sqrt{2}a\omega\cos\omega t_1$

$= 2\sqrt{2}a\omega.\left(-\frac{1}{\sqrt{2}}\right)$ (10)

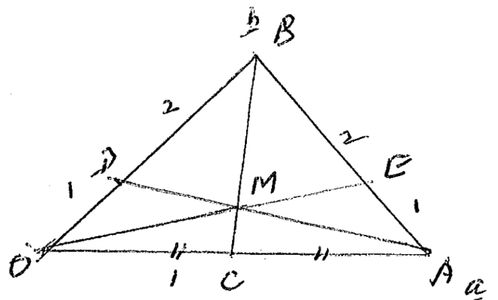
$= -2a\omega$

$A \rightarrow 0$ $v^2 = u^2 + 2as$

$v^2 = 4a^2\omega^2 - 2ga$

$v = u + at \Rightarrow t = \frac{2a\omega}{g} = \sqrt{\frac{2a}{g}}$ 10

14 a)



(10)

$\overrightarrow{BC} = \frac{a}{2} - b$ (5)

$\overrightarrow{DA} = a - \frac{1}{3}b$ (5)

$\overrightarrow{DC} = \frac{1}{2}a - \frac{1}{3}b$ (5)

$\overrightarrow{DM} = \lambda \overrightarrow{DA} = \lambda(a - \frac{1}{3}b)$ (5)

$\overrightarrow{MC} = \mu \overrightarrow{BC} = \mu(\frac{1}{2}a - b)$ (5)

$\overrightarrow{DM} + \overrightarrow{MC} = \overrightarrow{DC}$ (5)

$\lambda(a - \frac{1}{3}b) + \mu(\frac{1}{2}a - b) = \frac{1}{2}a - \frac{1}{3}b$

$(\lambda + \frac{1}{2}\mu - \frac{1}{2})a + (\frac{1}{3} - \frac{\lambda}{3} - \mu)b = 0$

$\lambda + \frac{1}{2}\mu - \frac{1}{2} = 0$ & $\frac{1}{3} - \frac{\lambda}{3} - \mu = 0$ (10)

$\lambda = \frac{2}{5}, \quad \mu = \frac{1}{5}$

$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$

$= \frac{1}{2}a - \frac{1}{5}(\frac{1}{2}a - b)$

$= \frac{2}{5}a + \frac{1}{5}b = \frac{1}{5}(2a + b)$ (10)

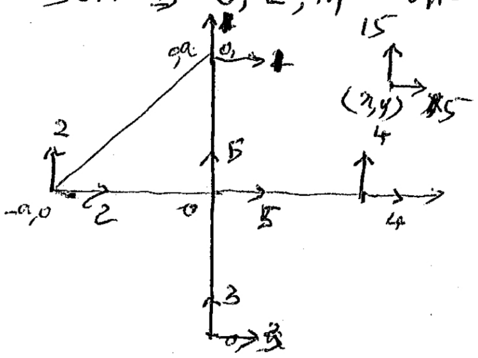
$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= b + \frac{2}{3}(a - b)$

$= \frac{2}{3}a + \frac{1}{3}b = \frac{1}{3}(2a + b)$

$= \frac{1}{3} \cdot 5\overrightarrow{OM}$ (10)

$3\overrightarrow{OE} = 5\overrightarrow{OM} \Rightarrow O, E, M$ collinear.

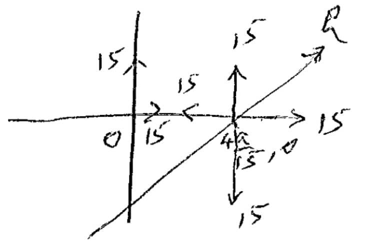


(10)

$R = 15(i + j)$ (10)

O: $15x - 15y = a.4 - a.1 - a.2 + a.3$ (10)

$15x - 15y = 4a$ (10)



$\equiv$ (10)

Couple to be added $G = 15 \cdot \frac{4a}{15}$ (10)

$= 4a$.

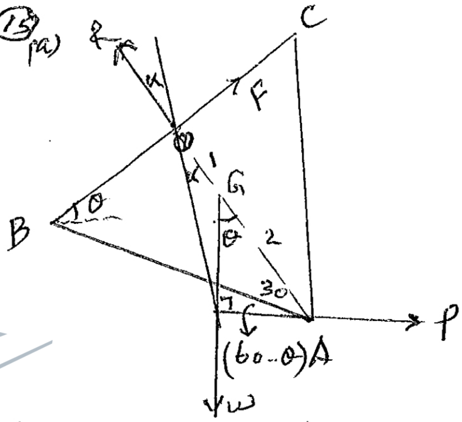
New Resultant $-15i + 15j$ (10)

line of action of new resultant

$15x + 15y = 4a$. (5)

Common Point is $(\frac{4a}{15}, 0)$ (5)

15 (a)



(20)

by Cot. Theorem (10)

$(1+2)\cot\theta = 1\cot(\theta - \alpha) - 2\cot 90$

$\tan\alpha = 3\tan(\theta - \alpha)$ (10)

$\tan\alpha = \frac{2\tan\theta}{3 + \tan^2\theta}$

for equilibrium $\alpha \le \lambda$ (5)

$\tan\alpha \le \tan\lambda$

$\frac{2\tan\theta}{3 + \tan^2\theta} \le \mu$, (5)

$\frac{2\tan\theta}{3 + \tan^2\theta} = y$ (say)

$y\tan^2\theta - 2\tan\theta + 3y = 0$ (5)

$\Delta \ge 0 \Rightarrow 4 - 12y^2 \ge 0$

$y \le \frac{1}{\sqrt{3}}$

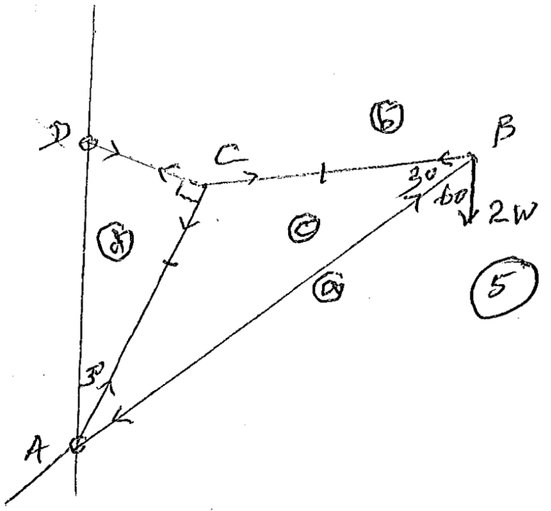
$y_{max} = \frac{1}{\sqrt{3}}$ (5)

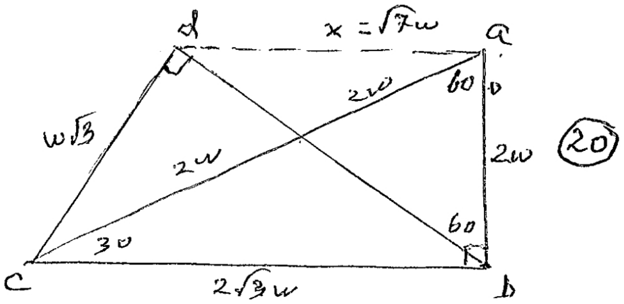
for equilibrium $y_{max} \le \mu$ (5)

$\frac{1}{\sqrt{3}} \le \mu$.

$\boxed{05}$

b)





Rod	Tension	Thrust
AB	–	4w
BC	$2\sqrt{3}w$	—
CD	3w	—
AC	$w\sqrt{3}$	—

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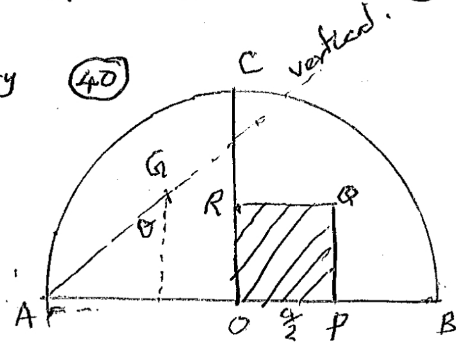
Reaction at D is 3w (10)

" at A is $\sqrt{7}w$ (10)

(16) Theory (40)



Mass	c.m from OC	from ⊥ A
$\frac{1}{2}\pi a^2\rho$	0	$\frac{4a}{3\pi}$
$\frac{a^2}{4}\rho$	$\frac{a}{4}$	$\frac{a}{4}$
$\frac{a^2\rho}{4}(2\pi-1)$	$\bar{x}$	$\bar{y}$

(45)

$\bar{x}\ \frac{a^2\rho}{4}(2\pi-1) = 0 \cdot \frac{1}{2}\pi a^2\rho - \frac{a^2}{4}\rho \cdot \frac{a}{4}$ (15)

$\bar{x} = -\frac{a}{4(2\pi-1)}$ (10)

$\bar{y} = \frac{29a}{12(2\pi-1)}$ equn – (15) (10)

$\tan\theta = \frac{a-\bar{x}}{\bar{y}}$ (15)

$= \frac{a - \frac{a}{4(2\pi-1)}}{\frac{29a}{12(2\pi-1)}}$

$= \frac{3(8\pi-5)}{29}$ (10)

(17) $P(A/B) = \frac{P(A\cap B)}{P(B)}$ (10)

$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$.

i $P(A'B'A) = P(A') \cdot P(B') \cdot P(A)$ (20)

$= \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{2}{5} = \frac{4}{25}$

ii $P(A'B'A'B) = \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{1}{3} = \frac{2}{25}$ (20)

(iii) $P(A'B'A'B' \dots B'A) = [P(A')]^{x-1}[P(B')]^{x-1} P(A)$ (20)

$= \left(\frac{3}{5}\right)^{x-1}\left(\frac{2}{3}\right)^{x-1} \cdot \frac{2}{5}$ (10)

$= \left(\frac{2}{5}\right)^{x-1} \cdot \frac{2}{5}$

$= \left(\frac{2}{5}\right)^{x}$ (10)

iv $P(A) + P(A'B'A) + P(A'B'A'B'A) \dots\dots$ (20)

$= \frac{2}{5} + \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{2}{5} + \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{2}{5}$ (10)

$= \frac{2}{5}\left[1 + \left(\frac{3}{5} \cdot \frac{2}{3}\right) + \left(\frac{3}{5} \cdot \frac{2}{3}\right)^2 + \dots\dots\right]$ (10)

$= \frac{2}{5}\left[1 + \frac{2}{5} + \left(\frac{2}{5}\right)^2 + \dots\right]$ (10)

$= \frac{2}{5}\left(\frac{1}{1-\frac{2}{5}}\right)$ (10)

$= \frac{2}{3}$

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