G.C.E. A/L Examination March - 2018

Conducted by Field Work Centre, Thondaimanaru

In Collaboration with

FWC

Grade :- 13 (2018)	Combined Mathematics - I	Marking Scheme

1. Let $f(n) = \frac{n^2}{2} + \frac{5n}{2}$.

When $n=1$, $f(1) = \frac{1}{2} + \frac{5}{2} = 3 \in \mathbb{Z}^+$

Hence the result is true for $n=1$. (5)

Assume that the result true for $n=p$

$f(p) = \frac{p^2}{2} + \frac{5p}{2} = k$; where $k \in \mathbb{Z}^+$ (5)

When $n = p+1$,

$f(p+1) = \frac{(p+1)^2}{2} + \frac{5(p+1)}{2}$ (5)

$= \frac{p^2}{2} + \frac{5p}{2} + p + 3$

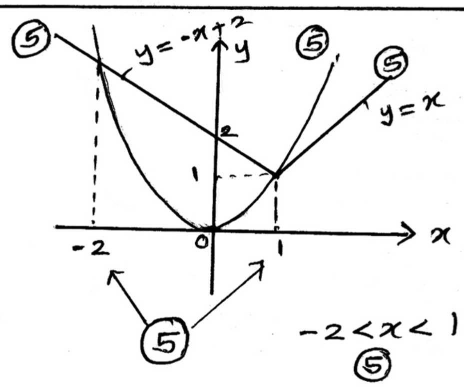
$= k + p + 3 \in \mathbb{Z}^+$

$\therefore$ the result is true for $n = p+1$ (5)

By the principle of mathematical Induction, the result is true for $n \in \mathbb{Z}^+$ (5)

$\boxed{25}$

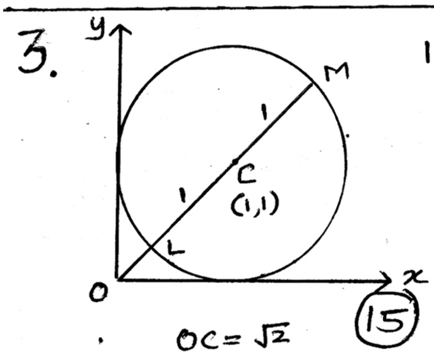
2.



$-2 < x < 1$ (5)

$\boxed{25}$

3.



$|z|_{max} = OM$
$= \sqrt{2} + 1$ (5)

$|z|_{min} = \sqrt{2} - 1$ (5)

$OC = \sqrt{2}$ (15)

$\boxed{25}$

4. (i) We have nine from whom we want to choose 3.

(5) ${}^9C_3 = \frac{9!}{6! \cdot 3!} = 84$ (5)

(ii) The number of selections containing at least one woman

= (total number of selection)
− (number of selection containg no woman)

$= {}^9C_3 - {}^4C_3$ (5) (5)

$= 84 - 4$

$= 80$ (5)

$\boxed{25}$

5. $\lim_{x \to 0} \frac{1 - \cos(2\sin x)}{x^2}$

$= \lim_{x \to 0} \frac{2\sin^2(\sin x)}{x^2}$ (5)

$= \lim_{x \to 0} 2 \frac{\sin^2(\sin x)}{\sin^2 x} \cdot \frac{\sin^2 x}{x^2}$ (5)

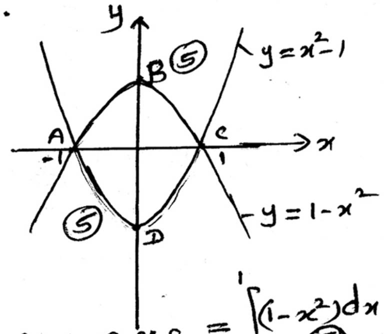
$= 2\left\{\lim_{\sin x \to 0} \frac{\sin(\sin x)}{\sin x}\right\}^2 \cdot \left\{\lim_{x \to 0} \frac{\sin x}{x}\right\}^2$ (5)

$= 2 \times 1^2 \times 1^2$ (5)

$= 2$ (5)

$\boxed{25}$

6.



Area of ABC $= \int_{-1}^{1} (1 - x^2)\,dx = \left[x - \frac{x^3}{3}\right]_{-1}^{1} = \frac{4}{3}$ (5) (5)

Area of ABCD $= 2 \times \frac{4}{3} = \frac{8}{3}$ square units. (5)

$\boxed{25}$

7. $x = \cos^2\theta + \sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta(-\sin\theta) + \cos\theta$ 5

$y = \sin^2\theta - \sin\theta$

$\frac{dy}{d\theta} = 2\sin\theta\cos\theta - \cos\theta$ 5

$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = \frac{\cos\theta(2\sin\theta - 1)}{\cos\theta(1 - 2\sin\theta)}$

$= -1$ 5

$P \equiv \left(\frac{5}{4}, -\frac{1}{4}\right)$ 5

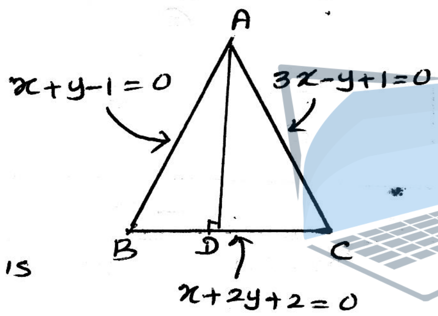
Equation of tangent at P:

$y + \frac{1}{4} = -1\left(x - \frac{5}{4}\right)$ 5

$x + y - 1 = 0$

$\boxed{25}$

8.



Equation of AD is

$x + y - 1 + \lambda(3x - y + 1) = 0.$ 5

$m_{AD} \times m_{BC} = -1$ 5

$\frac{1+3\lambda}{\lambda - 1} \times -\frac{1}{2} = -1 \Rightarrow \lambda = -3$ 5

5

AD: $2x - y + 1 = 0$ 5

$\boxed{25}$

9. $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$S' \equiv x^2 + y^2 - a^2 = 0$

Equation of common chord is $S - S' = 0$

$2gx + 2fy + c + a^2 = 0$ 5

$(0,0) \Rightarrow 0 + 0 + c + a^2 = 0$ 5

$\Rightarrow c = -a^2$

Since S passes through (1,2) 5

$1 + 4 + 2g + 4f + c = 0$

5 $5 + 2g + 4f - a^2 = 0$ $[\because c = -a^2]$

put $-g = x, \; -f = y$

$2x + 4y + a^2 - 5 = 0$ 5

$\boxed{25}$

10. $\tan(45^\circ + \theta) - \tan(45^\circ - \theta)$

$= \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ \tan\theta} - \frac{\tan 45^\circ - \tan\theta}{1 + \tan 45^\circ \tan\theta}$ 5

$= \frac{1 + \tan\theta}{1 - \tan\theta} - \frac{1 - \tan\theta}{1 + \tan\theta}$

$= \frac{(1+\tan\theta)^2 - (1-\tan\theta)^2}{1 - \tan^2\theta}$ 5

$= \frac{4\tan\theta}{1 - \tan^2\theta}$ 5

$= 2\tan 2\theta$

5

Put $\theta = 30^\circ$

$\tan 75^\circ - \tan 15^\circ = 2\tan 60^\circ$ 5

$\tan 75^\circ - \tan 15^\circ = 2\sqrt{3}$

$\boxed{25}$

11.

(a) $\alpha + \beta = a$ 5 $\quad \gamma + \delta = ac$ 5

$\alpha\beta = b \quad \gamma\delta = bc^2$

$\left(\frac{\alpha}{\gamma} + \frac{\beta}{\delta}\right) + \left(\frac{\alpha}{\delta} + \frac{\beta}{\gamma}\right)$

$= \frac{(\alpha+\beta)(\gamma+\delta)}{\gamma\delta}$ 10

$= \frac{a^2}{bc}$ 10

$\left(\frac{\alpha}{\gamma} + \frac{\beta}{\delta}\right)\left(\frac{\alpha}{\delta} + \frac{\beta}{\gamma}\right)$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\gamma\delta} + \alpha\beta\left[\frac{(\gamma+\delta)^2 - 2\gamma\delta}{(\gamma\delta)^2}\right]$ 10

$= \frac{a^2 - 2b}{bc^2} + \frac{a^2 - 2b}{bc^2}$ 10

$= \frac{2(a^2 - 2b)}{bc^2}$ 10

The eq^n whose roots are $\frac{\alpha}{\gamma} + \frac{\beta}{\delta}$ and $\frac{\alpha}{\delta} + \frac{\beta}{\gamma}$ is

$x^2 - \frac{a^2}{bc}x + \frac{2(a^2 - 2b)}{bc^2} = 0$ 10

$\Rightarrow bc^2x^2 - a^2cx + 2(a^2 - 2b) = 0$

$\Delta = a^4c^2 - 4bc^2 \, 2(a^2 - 2b)$ 10

$= c^2(a^2 - 4b)^2 \geq 0$ 10

$\boxed{90}$

$\therefore$ The roots are real

(b) $f(x) \equiv px^4 + qx^3 + 5x^2 - 6x + 2$

$f(1) = 0 \Rightarrow p + q + 5 - 6 + 2 = 0$ (5)

$p + q = -1$ (5) ①

$f(2) = f(-1)$

$16p + 8q + 20 - 12 + 2 = p - q + 5 + 6 + 2$ (5)

$\Rightarrow 5p + 3q = 1$ (5) ②

①, ② $\Rightarrow p = 2, \; q = -3$ (5) (5) $\boxed{30}$

$2x^4 - 3x^3 + 5x^2 - 6x + 2$

$\equiv (x-1)(2x^3 - x^2 + 4x - 2)$ (10)

$\equiv (x-1)\left[x^2(2x-1) + 2(2x-1)\right]$ (10)

$\equiv (x-1)(2x-1)(x^2+2)$

$f(x) = 0 \Rightarrow (x-1)(2x-1)(x^2+2) = 0$ (10)

$\Rightarrow x = 1$ or $x = \frac{1}{2}$ or $x = \pm\sqrt{2}\,i$ $\boxed{30}$

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12. (a) $U_r = \dfrac{(r+1)3^r}{(r+4)!}$ (10) $\boxed{10}$

$$\frac{(r+1)\,3^r}{(r+4)!} = \frac{k\,3^{r-1}}{(r+3)!} - \frac{k\,3^r}{(r+4)!} \quad (10)$$

$(r+1)3 = k(r+4) - k \cdot 3$ (5)

$(r+1)3 = (r+1)k$

$k = 3$ (5) $\boxed{20}$

$U_r = V_{r-1} - V_r$ (5)

$r=1 \quad U_1 = V_0 - \cancel{V_1}$

$r=2 \quad U_2 = \cancel{V_1} - \cancel{V_2}$

$\vdots$

$r=n-1 \quad U_{n-1} = \cancel{V_{n-2}} - \cancel{V_{n-1}}$ (5)

$r=n \quad U_n = \cancel{V_{n-1}} - V_n$

$\sum_{r=1}^{n} U_r = V_0 - V_n$ (5)

$= \dfrac{1}{8} - \dfrac{3^{n+1}}{(n+4)!}$ (10)

$\boxed{25}$

(b) $\left\{\dfrac{x(x+1)}{2}\right\}^2 - \left\{\dfrac{x(x-1)}{2}\right\}^2$

$\equiv \dfrac{x^2}{4}\left\{(x^2+2x+1) - (x^2-2x+1)\right\}$ (10)

$\equiv \dfrac{x^2}{4} \cdot 4x$ (5) $\boxed{15}$

$\equiv x^3$

$r^3 \equiv \left\{\dfrac{r(r+1)}{2}\right\}^2 - \left\{\dfrac{r(r-1)}{2}\right\}^2$

$r=1, \quad 1^3 = \left(\dfrac{1 \cdot 2}{2}\right)^2 - 0$ (5)

$r=2, \quad 2^3 = \left(\dfrac{2 \cdot 3}{2}\right)^2 - \left(\dfrac{2 \cdot 1}{2}\right)^2$

$\vdots$

$r=n-1, \quad (n-1)^3 = \left\{\dfrac{(n-1)n}{2}\right\}^2 - \left\{\dfrac{(n-1)(n-2)}{2}\right\}^2$

$r=n, \quad n^3 = \left\{\dfrac{n(n+1)}{2}\right\}^2 - \left\{\dfrac{n(n-1)}{2}\right\}^2$ (5)

$\sum_{r=1}^{n} r^3 = \dfrac{n^2(n+1)^2}{4}$ (5) $\boxed{15}$

(i) $1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3$

$= 1^3 + 2^3 + 3^3 + \cdots + (2n)^3 - \{2^3 + 4^3 + \cdots + (2n)^3\}$ (10)

$= \dfrac{(2n)^2(2n+1)^2}{4} - 8\{1^3 + 2^3 + \cdots + n^3\}$ (10)

$= n^2(2n+1)^2 - \dfrac{8n^2(n+1)^2}{4}$ (5)

$= n^2\{4n^2 + 4n + 1 - 2(n^2 + 2n + 1)\}$ (10)

$= n^2(2n^2 - 1)$ $\boxed{35}$

(ii) $n^3 + (n+1)^3 + \cdots + (2n)^3$

$= 1^3 + 2^3 + 3^3 + \cdots + (2n)^3 - \{1^3 + 2^3 + \cdots + (n-1)^3\}$ (10)

$= \dfrac{4n^2(2n+1)^2}{4} - \dfrac{(n-1)^2 n^2}{4}$ (10)

$= \dfrac{3n^2}{4}(5n^2 + 6n + 1)$

$= \dfrac{3n^2(n+1)(5n+1)}{4}$ (10) $\boxed{30}$

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13. (a) (i) $z_1 = \dfrac{2\sqrt{2}}{1-i}$

$= \dfrac{2\sqrt{2}(1+i)}{2}$ (5)

$= 2\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$ (5)

$z_1 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ (5)

$|z_1| = 2$, (5) $\quad \operatorname{Arg} z_1 = \frac{\pi}{4}$ (5)

$\boxed{25}$

$$z_2 = \frac{2(1+\sqrt{3}i)}{\sqrt{3}+i}$$

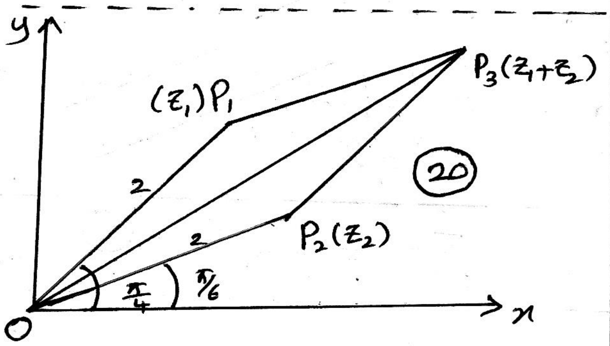
$$= \frac{2(1+\sqrt{3}i)(\sqrt{3}-i)}{3+1} \quad (5)$$

$$= 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) \quad (5)$$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \quad (5)$$

$|z_2| = 2$, $\quad \mathrm{Arg}(z_2) = \frac{\pi}{6}$

(5) (5) $\boxed{25}$



$OP_1 = OP_2 = 2$

$\therefore \square\, OP_1P_3P_2$ is a rhombus (5)

$P_1\hat{O}P_3 = P_2\hat{O}P_3 = \frac{\pi}{24}$ (5)

$x\hat{O}P_3 = \frac{\pi}{6} + \frac{\pi}{24} = \frac{5\pi}{24}$ (5)

$\mathrm{Arg}(z_1+z_2) = \frac{5\pi}{24}$ $\boxed{35}$

$$z_1 + z_2 = \sqrt{2}(1+i) + \sqrt{3} + i$$

$$= (\sqrt{3}+\sqrt{2}) + i(\sqrt{2}+1) \quad (5)$$

$$\tan\frac{5\pi}{24} = \frac{\sqrt{2}+1}{\sqrt{3}+\sqrt{2}} \quad (5)$$

$$= \frac{(\sqrt{2}+1)(\sqrt{3}-\sqrt{2})}{1} \quad (5)$$

$$= \sqrt{6} + \sqrt{3} - \sqrt{2} - 2 \quad \boxed{15}$$

$$P_1P_2 = |z_2 - z_1| \quad (5)$$

$$= |\sqrt{3}+i - \sqrt{2}(1+i)|$$

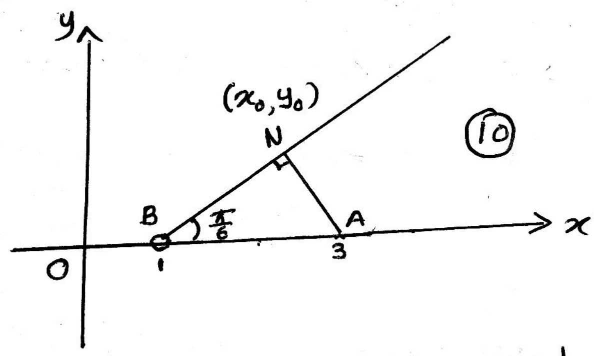
$$= |(\sqrt{3}-\sqrt{2}) + i(1-\sqrt{2})|$$

$$= \sqrt{(\sqrt{3}-\sqrt{2})^2 + (\sqrt{2}-1)^2} \quad (5)$$

$$= 8 - 2\sqrt{6} - 2\sqrt{2}$$

$\boxed{15}$

(b)



$|z-3|_{min} = AN = 2\sin\frac{\pi}{6} = 1$ unit

(5) (5)

Let $N \equiv (x_0, y_0)$

$$x_0 = 1 + 2\cos^2\frac{\pi}{6} = \frac{5}{2} \quad (5)$$

$$y_0 = 2\cos\frac{\pi}{6}\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad (5)$$

Hene the required complex number:

$z_0 = \frac{5}{2} + i\frac{\sqrt{3}}{2}$ (5) $\boxed{35}$

14.

(a) $$f(x) = \frac{x(2x+1)}{(x-1)(2x+3)}$$

$$f'(x) = \frac{(x-1)(2x+3)(4x+1) - x(2x+1)(4x+1)}{(x-1)^2(2x+3)^2} \quad (10)$$

$$= \frac{(4x+1)(2x^2+x-3-2x^2-x)}{(x-1)^2(2x+3)^2} \quad (10)$$

$$= -\frac{3(4x+1)}{(x-1)^2(2x+3)^2} \quad \boxed{20}$$

Vertical asymptotes: $x = -\frac{3}{2}$ and $x = 1$

(5) (5)

Horizontal asymptote:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{1\left(2+\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)\left(2+\frac{3}{x}\right)} = 1$$

Hence, it is $y = 1$ (5) $\boxed{15}$

$f'(x) = 0 \Rightarrow x = -\frac{1}{4}$ (5)

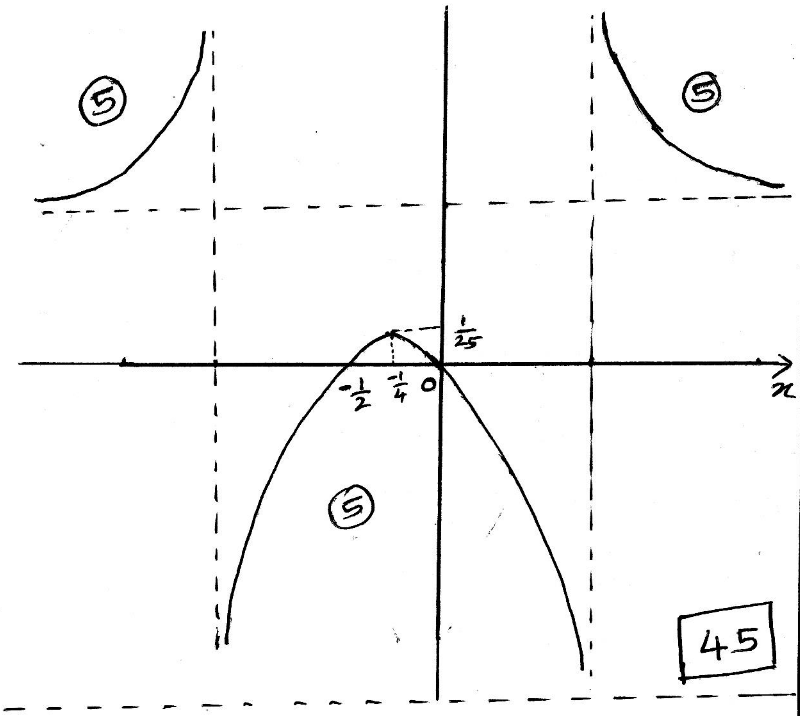
	$x < -\frac{3}{2}$	$-\frac{3}{2} < x < -\frac{1}{4}$	$-\frac{1}{4} < x < 1$	$x > 1$
$f'(x)$	$(+)$	$(+)$	$(-)$	$(-)$

(5) (5) (5) (5)

$y = f(x)$ has only one turning point;

$\left(-\frac{1}{4}, \frac{1}{25}\right)$ - local maximum (5)

$f(x) = 0 \Rightarrow x = 0$ or $x = -\frac{1}{2}$



$\boxed{45}$

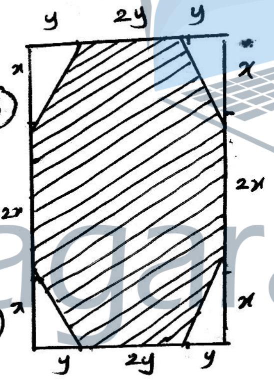
$$0 \le \frac{x(2x+1)}{(x-1)(2x+3)} \le \frac{1}{25}$$

$$\Rightarrow 0 \le f(x) \le \frac{1}{25}$$

$$\Rightarrow -\frac{1}{2} \le x \le 0 \qquad (10)$$

$\boxed{10}$

(b)



$\text{Area} = (4y)(4x) - 4\left(\frac{1}{2}xy\right)$ (5)

$14 = 16xy - 2xy$

$y = \frac{1}{x}$ (5)

$P = 4y + 4x + 4\sqrt{x^2+y^2}$ (5)

$= 4\left(\frac{1}{x} + x + \sqrt{x^2 + \frac{1}{x^2}}\right)$

$= \frac{4}{x}\left(1 + x^2 + \sqrt{x^4+1}\right)$ (5)

$$\frac{dP}{dx} = \frac{x \cdot 4\left(2x + \frac{1}{2\sqrt{x^4+1}}4x^3\right) - \left(1+x^2+\sqrt{x^4+1}\right)4}{x^2} \qquad (10)$$

$$= \frac{4(x^2-1)\left(\sqrt{x^4+1} + x^2+1\right)}{x^2\sqrt{x^4+1}} \qquad (10)$$

$\frac{dP}{dx} = 0 \Rightarrow x = 1 \quad \because x > 0$ (5)

$\frac{dP}{dx} < 0$ for $0 < x < 1$ (5)

$\frac{dP}{dx} > 0$ for $x > 1$ (5)

$\therefore P$ is minimum when $x = 1$ (5)

$\boxed{60}$

15. (a) $\displaystyle\int_7^{13} \frac{1}{(x-1)\sqrt{4x-3}}\,dx$

Let $t = \sqrt{4x-3}$. (5)

Then $t^2 = 4x - 3$ (5)

$2t\frac{dt}{dx} = 4$

When $x = 7$, $t = 5$ (5)

When $x = 13$, $t = 7$

$$\int_7^{13} \frac{1}{(x-1)\sqrt{4x-3}}\,dx = \int_5^7 \frac{\frac{t}{2}\,dt}{\frac{t^2-1}{4}\cdot t} \qquad (10)$$

$$= \int_5^7 \frac{2\,dt}{t^2-1}$$

$$= \int_5^7 \frac{1}{t-1}\,dt - \int_5^7 \frac{1}{t+1}\,dt \qquad (10)$$

$$= \Big[\ln|t-1| - \ln|t+1|\Big]_5^7 \qquad (10)$$

$$= \ln\frac{6}{8} - \ln\frac{4}{6} \qquad (5)$$

$$= \ln\frac{9}{8}$$

$\boxed{50}$

(b) $\dfrac{1}{(x^2-1)(x^2+3x+2)} = \dfrac{1}{(x-1)(x+1)^2(x+2)}$ (5)

$$\frac{1}{(x-1)(x+1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{x+2} \qquad (5)$$

$$1 \equiv A(x+1)^2(x+2) + B(x-1)(x+1)(x+2) + C(x-1)(x+2) + D(x-1)(x+1)^2$$

$x = 1 \Rightarrow 1 = 12A \Rightarrow A = \frac{1}{12}$ (5)

$x = -1 \Rightarrow 1 = -2C \Rightarrow C = -\frac{1}{2}$ (5)

$x = -2 \Rightarrow 1 = -3D \Rightarrow D = -\frac{1}{3}$ (5)

$x^3 /\!/ \quad 0 = A + B + D \Rightarrow \quad = \frac{1}{4}$ (5)

$$\frac{1}{(x^2-1)(x^2+3x+2)} = \frac{\frac{1}{12}}{x-1} + \frac{\frac{1}{4}}{x+1} - \frac{\frac{1}{2}}{(x+1)^2} - \frac{\frac{1}{3}}{x+2}$$

$$\int \frac{1}{(x^2-1)(x^2+3x+2)}\,dx = \int \frac{\frac{1}{12}}{x-1}\,dx + \int \frac{\frac{1}{4}}{x+1}\,dx - \int \frac{\frac{1}{2}}{(x+1)^2}\,dx - \int \frac{\frac{1}{3}}{(x+2)}\,dx$$

$$= \frac{1}{12}\ln|x-1| + \frac{1}{4}\ln|x+1| + \frac{1}{2(x+1)} - \frac{1}{3}\ln|x+2| + C$$

(5) (5) (5) (5)

$\boxed{50}$

(c) $I = \int_1^{e^\pi} \sin(\ln x)\,dx$

$= \Big[\sin(\ln x)\,x\Big]_1^{e^\pi} - \int_1^{e^\pi} x\cos(\ln x)\frac{1}{x}\,dx$ 10

$I = 0 - J$ 5

$I = -J$ —— (1) $\boxed{15}$

$J = \int_1^{e^\pi} \cos(\ln x)\,dx$

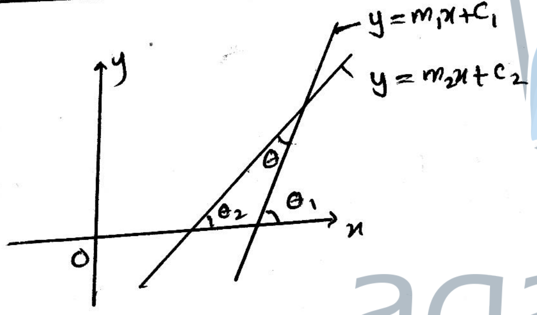
$= \Big[\cos(\ln x)\,x\Big]_1^{e^\pi} - \int_1^{e^\pi} x(-\sin(\ln x))\frac{1}{x}\,dx$ 10

$= -e^\pi - 1 + I$ 5

$J - I = -(e^\pi + 1)$ —— (2) $\boxed{15}$

$I = \frac{1}{2}(e^\pi + 1)$ 10 , $J = -\frac{1}{2}(e^\pi+1)$ 10 $\boxed{20}$

16.



$\theta = \theta_1 - \theta_2$ 5

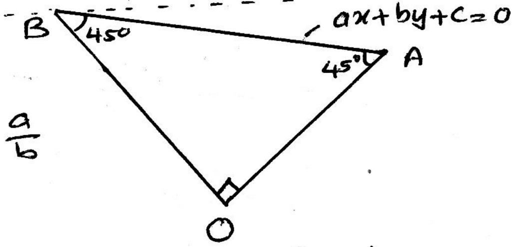
$\tan\theta = \tan(\theta_1 - \theta_2)$

$= \dfrac{\tan\theta_1 - \tan\theta_2}{1+\tan\theta_1\tan\theta_2}$ 5

$= \dfrac{m_1 - m_2}{1 + m_1 m_2}$

Since θ is an acute angle 5 $\boxed{15}$

$\therefore \tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$



$m_{AB} = -\frac{a}{b}$

$\tan 45^0 = \left|\dfrac{m - (-\frac{a}{b})}{1 + m(-\frac{a}{b})}\right|$ 10

$1 = \left|\dfrac{mb+a}{b-ma}\right|$ 5

$\dfrac{mb+a}{b-ma} = \pm 1$ 5

$(+) \Rightarrow m = \dfrac{b-a}{b+a}$ 5 $(-) \Rightarrow m = \dfrac{a+b}{a-b}$ 5

Equations: $y = -\left(\dfrac{a-b}{a+b}\right)x$ 5 or $y = \left(\dfrac{a+b}{a-b}\right)x$ 5

or

$(a-b)x + (a+b)y = 0$ or $(a+b)x - (a-b)y = 0$ $\boxed{40}$

(b) $S \equiv x^2+y^2+2gx+2fy+c=0$

$(2,0) \Rightarrow 4+0+4g+0+c=0$

$\Rightarrow 4+4g+c=0$ —— (1) 5

$(0,2) \Rightarrow 0+4+0+4f+c=0$

$\Rightarrow 4+4f+c=0$ —— (2) 5

$(1)-(2) \Rightarrow g = f$ 5

$c = -4-4g$ 5

General equation

$x^2+y^2+2gx+2gy-4-4g=0$ 5 $\boxed{25}$

$S' \equiv x^2+y^2-4x+6y-10=0$

Equation of the common chord $S-S'=0$

$(2g+4)x+(2g-6)y+(6-4g)=0$ 10

$(-g,-g) \Rightarrow (2g+4)(-g)+(2g-6)(-g)+6-4g=0$ 10
5

$\Rightarrow 2g^2+g-3=0$ 5

$\Rightarrow (2g+3)(g-1)=0$ 5

$\Rightarrow g = -\frac{3}{2}$ or $g=1$ 5

$g=-\frac{3}{2} \Rightarrow x^2+y^2-3x-3y+2=0$ 5

$g=1 \Rightarrow x^2+y^2+2x+2y-8=0$ 5 $\boxed{50}$

$2g_1g_2+2f_1f_2$

$=2(-\frac{3}{2})(1)+2(-\frac{3}{2})(1)$ 10

$=-6$ 5

$=2+(-8)$ 5 $\boxed{20}$

$=c_1+c_2$

$\therefore$ Two circles intersect orthogonally.

17.

(a) $\sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$ (5)

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$ (10)

$= 1.(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$ (5)

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$ (5)

$= 1 - 3\sin^2\theta\cos^2\theta$ $\boxed{25}$

$\sin^6\theta + \cos^6\theta + \sin\theta\cos\theta = 1$

$1 - 3\sin^2\theta\cos^2\theta + \sin\theta\cos\theta = 1$ (5)

$\sin\theta\cos\theta(1 - 3\sin\theta\cos\theta) = 0$ (5)

$\sin\theta\cos\theta = 0$ (5)

$\Rightarrow \sin 2\theta = 0 = \sin 0$

$\Rightarrow 2\theta = n\pi, \; n \in \mathbb{Z}$

$\Rightarrow \theta = \frac{n\pi}{2}, \; n \in \mathbb{Z}$ (5)

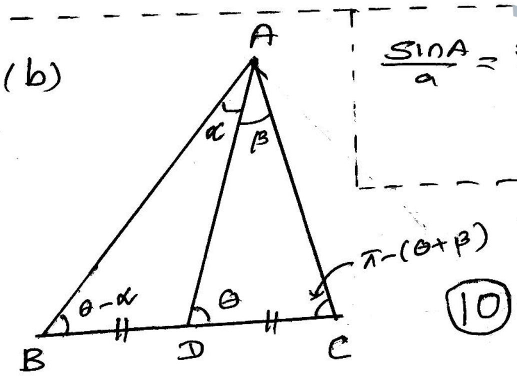
or $\sin\theta\cos\theta = \frac{1}{3}$

$\sin 2\theta = \frac{2}{3} = \sin\alpha$; where $\alpha = \sin^{-1}\left(\frac{2}{3}\right)$ (5)

$2\theta = n\pi + (-1)^n\alpha$; $n \in \mathbb{Z}$

$\theta = \frac{n\pi}{2} + (-1)^n\frac{\alpha}{2}$; $n \in \mathbb{Z}$ $\boxed{30}$ (5)

(b)



(10)

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (5) $\boxed{5}$

$\triangle ABD$

$\frac{BD}{\sin\alpha} = \frac{AD}{\sin(\theta - \alpha)}$ (10)

$\Rightarrow AD = \frac{BD\sin(\theta - \alpha)}{\sin\alpha}$ ——— (1)

$\triangle ADC$

$\frac{DC}{\sin\beta} = \frac{AD}{\sin(\pi - (\theta + \beta))}$ (10)

$AD = \frac{DC\sin(\theta + \beta)}{\sin\beta}$ ——— (2)

(1), (2) $\Rightarrow$

$\frac{BD\sin(\theta - \alpha)}{\sin\alpha} = \frac{DC\sin(\theta + \beta)}{\sin\beta}$ (5)

$\Rightarrow \sin\beta\sin(\theta - \alpha) = \sin\alpha\sin(\theta + \beta)$

$\Rightarrow \sin\beta(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ (10)

$= \sin\alpha(\sin\theta\cos\beta + \cos\theta\sin\beta)$

$\Rightarrow 2\sin\alpha\sin\beta\cos\theta = \sin\theta\sin\beta\cos\alpha - \sin\theta\cos\beta\sin\alpha$ (5)

$\Rightarrow 2\cot\theta = \cot\alpha - \cot\beta$ (5)

$\boxed{55}$

(c) $\sin^{-1}\left(\frac{24}{x}\right) + \sin^{-1}\left(\frac{7}{x}\right) = \frac{\pi}{2}$

Let $\alpha = \sin^{-1}\left(\frac{24}{x}\right)$, $\beta = \sin^{-1}\left(\frac{7}{x}\right)$ (5)

$\alpha + \beta = \frac{\pi}{2}$ (5)

$\Rightarrow \alpha = \frac{\pi}{2} - \beta$

$\sin\alpha = \sin\left(\frac{\pi}{2} - \beta\right)$ (5)

$\sin\alpha = \cos\beta$ (5)

$\sin^2\alpha = \cos^2\beta = 1 - \sin^2\beta$

$\frac{576}{x^2} = 1 - \frac{49}{x^2}$ (5)

$\Rightarrow x^2 = 25$ (5) $[\because x > 0]$

$\Rightarrow x = 5$ (5)

$\boxed{35}$

G.C.E. A/L Examination March - 2018

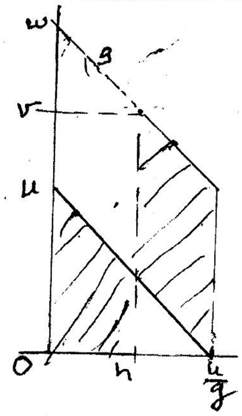
Conducted by Field Work Centre, Thondaimanaru

In Collaboration with

FWC

Grade :- 13 (2018)	Combined Mathematics - II	Marking scheme

(1)



(10)

$\frac{w-v}{h} = g \Rightarrow w = v + gh$ (5)

$vh = (w-u)\frac{u}{g} - \frac{1}{2}(w-v)h$ (5)

$vh = (v+gh-u)\frac{u}{g} - \frac{h}{2}(gh)$

$v(\cancel{h} - \frac{u}{g}) = uh - \frac{u^2}{g} - \frac{gh^2}{2}$

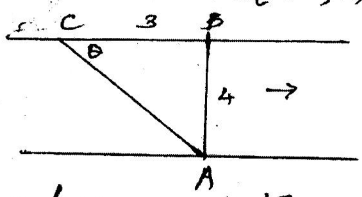
$v(gh-u) = guh - u^2 - \frac{g^2h^2}{2}$

$= u(gh-u) - \frac{g^2h^2}{2}$ (5)

$v = u - \frac{g^2h^2}{2(gh-u)}$

$= u + \frac{g^2h^2}{2(u-gh)}$

(2)

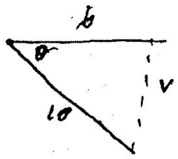


(5)

$V_{RE} = 6 \rightarrow$ $\quad V_{MR}$ 10 $\quad V_{ME}$ (v)

$V_{ME} = V_{MR} + V_{RE}$ (5)

10 $\quad \overrightarrow{6}$



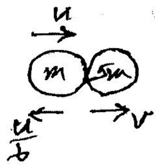
$\cos\theta = \frac{3}{5}$ (5)

$v^2 = 10^2 + 6^2 - 2.10.6.\cos\theta$

$v = 8$ (5)

Man arrived at B (5)

(3)



$\overrightarrow{I} = \Delta mv$

$0 = (5mv - m.\frac{u}{6}) - mu$ (10)

$v = \frac{7u}{30}$ (5)

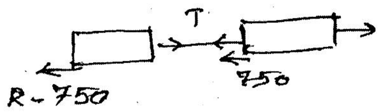
N.R. Law (5)

$v + \frac{u}{6} = e.u$ (5)

$\frac{7u}{30} + \frac{u}{6} = eu$

$e = \frac{12}{30} = \frac{2}{5}$

(4)



(5)

$P = F.v$

$50\times10^3 = F.25$

$F = 2000N$ (5)

for Car + Trailer applying $F = ma$

$F - R = 0$

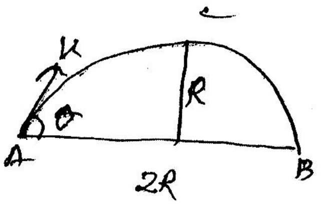
$R = F = 2000N$ (5)

for Trailer

$T - (R-750) = 0$ (5)

$T = 1250\ N$ (5)

(5)



$A \to B$ $\rightarrow$ $2R = u\cos\theta \cdot t$ (5)

$\uparrow$ $0 = u\sin\theta \cdot t - \frac{1}{2}gt^2$ (5)

$t = \frac{2u\sin\theta}{g}$

$2R = u\cos\theta \cdot \frac{2u\sin\theta}{g}$ (5)

$R = \frac{u^2\sin\theta\cos\theta}{g}$ — (1)

$\uparrow A \to C \Rightarrow 0 = u^2\sin^2\theta - 2gR$

$R = \frac{u^2\sin^2\theta}{2g}$ — (2) (5)

(1)+(2) $\Rightarrow$ $\sin\theta = 2\cos\theta$

$\tan\theta = 2$

$\theta = \tan^{-1} 2$

(2) $\Rightarrow R = \frac{u^2}{2g} \cdot \frac{4}{5}$ $\qquad \sin\theta = \frac{2}{\sqrt{5}}$

$= \frac{2u^2}{5g}$

$u^2 = \frac{5gR}{2}$

$u = \sqrt{\frac{5gR}{2}}$ (5)

(6)

$\underline{b} = \lambda\underline{a} + 2\underline{c}$ $\qquad \tan\theta = \sqrt{15}$

$b \cdot b = (\lambda\underline{a} + 2\underline{c}) \cdot (\lambda\underline{a} + 2\underline{c})$ (5)

$b^2 = \lambda^2 a^2 + 4c^2 + 4\lambda\, \underline{a}\cdot\underline{c}$ (5)

$4^2 = \lambda^2 \cdot 1 + 4\cdot 1 + 4\lambda\, ac\cos\theta$ (5)

$12 = \lambda^2 + 4\lambda \cdot 1 \cdot 1 \cdot \frac{1}{4}$ (5)

$\lambda^2 + \lambda - 12 = 0$

$(\lambda + 4)(\lambda - 3) = 0$

$\lambda = -4, 3$ (5)

(7)

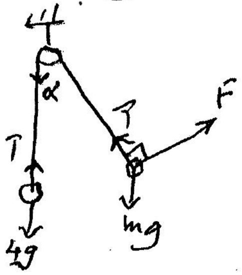
$\uparrow T = 4g$ (5)

$\uparrow T\cos\alpha + F\sin\alpha = mg$ (5)

$\rightarrow T\sin\alpha = F\cos\alpha$ (5)

$4g\tan\alpha = F$

$4g\,\frac{3}{4} = F \Rightarrow F = 3g$ (5)

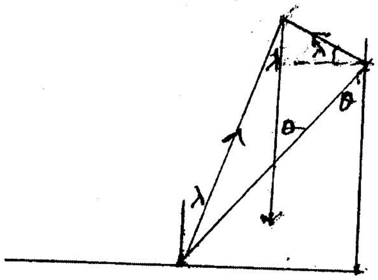


$4g\,\frac{4}{5} + 3g \cdot \frac{3}{5} = mg$

$5g = mg$

$m = 5$ (5)

(8)



(10)

$2\cot\theta = \cot\lambda - \cot(90 - \lambda)$ (5)

$= \cot\lambda - \tan\lambda$

$\frac{2}{\tan\theta} = \frac{1}{\tan\lambda} - \tan\lambda$

$\frac{2}{\tan\theta} = \frac{1 - \tan^2\lambda}{\tan\lambda}$ (5)

$\tan\theta = \frac{2\tan\lambda}{1 - \tan^2\lambda}$

$= \tan 2\lambda$ (5)

$\theta = 2\lambda.$

(9) $\frac{1+3p}{2} + \frac{1-4p}{3} + \frac{1+p}{6} = 1$ (10)

$3(1+3p) + 2(1-4p) + 1 + p = 6$ (5)

$2p + 7 = 6$ (5)

$p = \frac{1}{2}$ (5)

(10) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ (5)

$\frac{3}{8} = \frac{P(A \cap B)}{\frac{1}{3}}$

$P(A \cap B) = \frac{1}{8}$ (5)

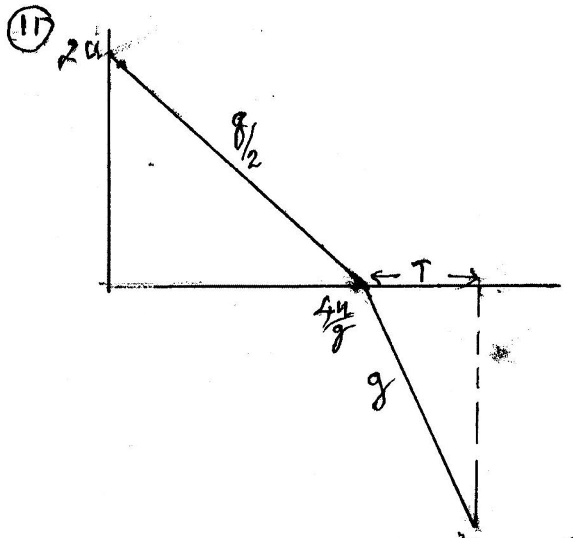
$P(A' \cap B') = P(A \cup B)'$ (5)

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$ (5)

$= \frac{11}{24}$ (5)

(11)



(10)

Start length $= \frac{1}{2} \cdot 2u \cdot \frac{4u}{g} = \frac{4u^2}{g}$ (10)

Vertical height $= \frac{4u^2}{g} \sin 30 = \frac{2u^2}{g}$ (10)

$\frac{1}{2} g T^2 = \frac{2u^2}{g}$ (10)

$T = \frac{2u}{g}$

Total time $= \frac{4u}{g} + \frac{2u}{g} = \frac{6u}{g}$ (10)

Velocity $= gT = 2u$. (10)

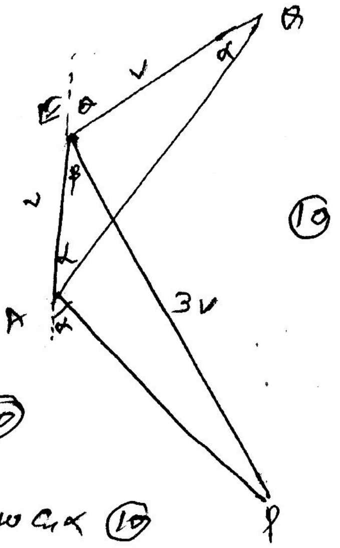
$\uparrow \; v^2 = u^2 + 2as$

$0 = (e.2u)^2 - 2g \cdot \frac{u^2}{g}$ (10)

$e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$ (10)

b) $V_{AE} = \uparrow v$, $V_{PE} = 3v$, V_{PA} (10)

$V_{QE} = v$ V_{QA} (10)



(10)

$AQ = 2v \cos\alpha$ (10)

$AP = w$

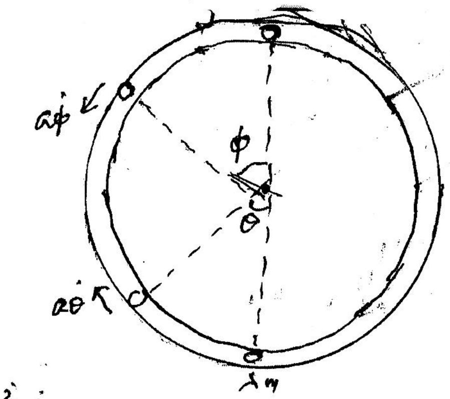
$9v^2 = w^2 + v^2 + 2vw \cos\alpha$ (10)

$w = v\sqrt{8 + \cos^2\alpha} - v\cos\alpha$

Q moves in $S\theta^\circ W$, $\theta = 2\alpha$ (10)

P moves in $N\beta^\circ W$, $\beta = \sin^{-1}\left(\frac{w \sin\alpha}{3v}\right)$ (10)

12).



(10)

for m $\frac{1}{2} m (a\dot{\phi})^2 + mga\cos\phi = mga$ (10)

$a\dot{\phi}^2 = 2g(1 - \cos\phi)$ (5)

$a\dot{\phi} = 2\sqrt{ag} \sin\frac{\phi}{2}$

for λm

$\frac{1}{2} \lambda m (a\dot{\theta})^2 - \lambda mga\cos\theta = \frac{1}{2}\lambda m u^2 - \lambda mga$ (10)

$(a\dot{\theta})^2 = u^2 - 2ag(1 - \cos\theta)$. (5)

$\theta = \frac{\pi}{2} \Rightarrow a\dot{\theta} = \sqrt{u^2 - 2ag}$ (10)

$\phi = \frac{\pi}{2} \Rightarrow a\dot{\phi} = \sqrt{2ag}$ (10)

$\Rightarrow 2ag = u^2 - 2ag$.

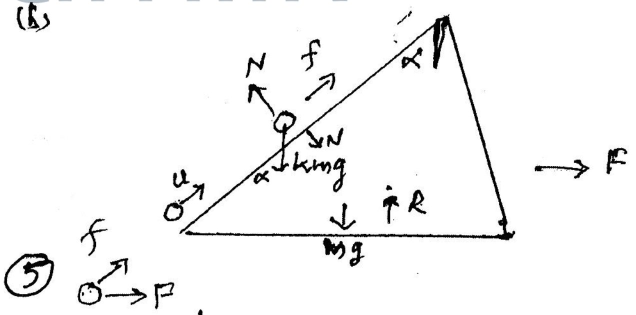
$u = 2\sqrt{ag}$. (10)

Let v be the velocity after the impact

$(\lambda + 1) m v = \lambda m \sqrt{u^2 - 2ag} - m\sqrt{2ag}$

$v = \frac{\lambda\sqrt{u^2 - 2ag} - \sqrt{2ag}}{\lambda + 1}$. (10)

(b)



(10)

(5)

For the system $F = ma \rightarrow$

$0 = MF + \lambda m (F + f\sin\alpha)$ (15)

$(1+\lambda) F = -\lambda f \sin\alpha$ — (1)

for $\lambda m \nearrow$

$-\lambda mg\cos\alpha = \lambda m (f + F\sin\alpha)$ — (2) (15)

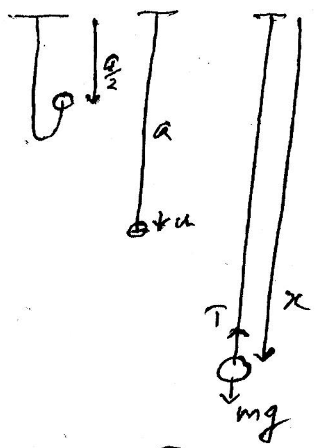
$f = \frac{-(\lambda+1) g \cos\alpha}{1 + k\cos^2\alpha}$ (10)

form $\nearrow$ $v^2 = u^2 + 2as$

$0 = u^2 + 2f \cdot AB$ (10)

$AB = \frac{u^2 (1 + k\cos^2\alpha)}{2(1+k) g\cos\alpha}$ (5)

(13)



(10)

$u = \sqrt{2ag}$ (10)

for m ↓ $mg - T = m\ddot{x}$

$T = \frac{mg}{a}(x-a)$ (10)

$mg - \frac{mg}{a}(x-a) = m\ddot{x}$

$\ddot{x} + \frac{g}{a}(x-2a) = 0$ —— (1) (10)

$x - 2a = A\cos\omega t + B\sin\omega t$ —— (2)

$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$ —— (3)

(10) $\ddot{x} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$

$= -\omega^2(x-2a)$

(10) $= -\frac{g}{a}(x-2a)$ —— (1)

⇒ x is a sol^n of (1) (5)

when t = 0, x = a, $\dot{x} = \sqrt{2ag}$ (5)

(2) ⇒ $-a = A\cos 0 + B\sin 0$

$A = -a$ (10)

(3) ⇒ $B = a$

∴ $x - 2a = a(\sin\omega t - \cos\omega t)$

$\dot{x} = a\omega(\cos\omega t + \sin\omega t)$ (10)

$x = 2a \Rightarrow \sin\omega t - \cos\omega t = 0$ (10)

$\tan\omega t = 1$

$\omega t = \frac{\pi}{4}$

$t = \frac{\pi}{4\omega}$ (10)

⇒ $x = 2a$, $\dot{x} = a\omega\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)$ (10)

$= a\sqrt{\frac{g}{a}}\cdot\frac{1}{\sqrt{2}}\cdot 2$

$= \sqrt{2ag}$ (10)

$\dot{x} = 0 \Rightarrow \tan\omega t = -1$

$\omega t = \frac{3\pi}{4}$ (∵ t > 0)

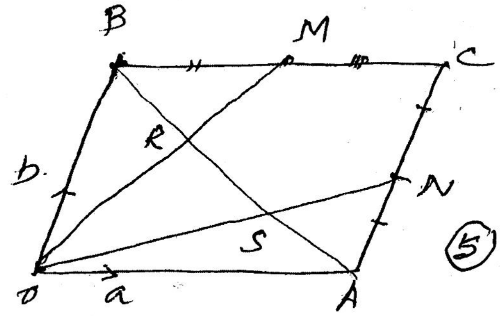
$t = \frac{3\pi}{4\omega}$ (10)

$\dot{x} = 0 \Rightarrow x = 2a + a\left(\sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}\right)$ (5)

$= 2a + a\cdot\frac{1}{\sqrt{2}}\cdot 2 = 2a + \sqrt{2}a$

Amplitude = $\sqrt{2}a$ (5)

(14)



(5)

$\vec{ON} = n = \vec{OA} + \vec{AN}$ (5)

$= a + \frac{1}{2}b$

$\vec{OM} = m = \vec{OB} + \vec{BM}$ (5)

$= b + \frac{1}{2}a$

$\vec{NM} = m - n$

$= \frac{1}{2}b - \frac{1}{2}a$ (5)

$= \frac{1}{2}(b - a)$

$= \frac{1}{2}\vec{AB}$ (5)

∴ NM // AB & NM = $\frac{1}{2}$AB (5)

$\vec{OS} = \lambda\vec{ON}$ $\quad \vec{AS} = \mu\vec{AB}$ (10)

$\vec{OS} + \vec{SA} = \vec{OA}$

$\lambda(a + \frac{1}{2}b) - \mu(b - a) = a$ (10)

$(\lambda + \mu - 1)a + (\frac{\lambda}{2} - \mu)b = 0$

$\lambda + \mu - 1 = 0$ & $\frac{\lambda}{2} - \mu = 0$ (10)

$\lambda = \frac{2}{3}$, $\mu = \frac{1}{3}$

∴ $\vec{OS} = \frac{2}{3}\vec{ON} = \frac{2}{3}(a + \frac{1}{2}b)$

$= \frac{2}{3}a + \frac{1}{3}b$ (5)

Illy $OR = \frac{2}{3}b + \frac{1}{3}a$ (5)

$\vec{AS} = \frac{1}{3}\vec{AB}$

$\vec{SR} = r - s$

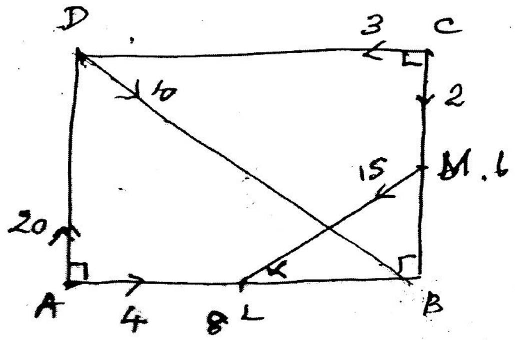
$= \frac{2}{3}b + \frac{1}{3}a - (\frac{2}{3}a + \frac{1}{3}b)$

$= \frac{1}{3}(b - a)$

$= \frac{1}{3}AB$

∴ AS : SR : RB = 1 : 1 : 1 (5)

(14) b)



(10)

$\overrightarrow{AB}$ X

$\tan\alpha = \frac{3}{4}$

$X = 4 - 3 + 10\cos\alpha - 15\cos\alpha$ (10)

$= 1 - 5.\frac{4}{5}$

$= -3$ (5)

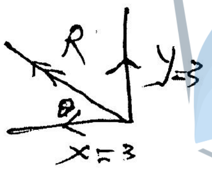
$\leftarrow = 3$

$\uparrow AD,\ Y = 20 - 2 - 10\sin\alpha - 15\sin\alpha$ (10)

$= 18 - 25.\frac{3}{5}$

$= 3$

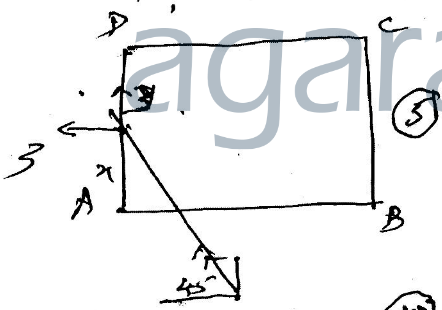
(5)



(5)

$R = 3\sqrt{2}$

$\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$ (5)



(5)

(10)

$A\circlearrowleft \quad x.3 = 3.6 - 2.8 - 15.4\sin\alpha - 10.8\sin\alpha$

$= 18 - 16 - 15.4.\frac{3}{5} - 10.8.\frac{3}{5}$

$= -82$

$x = -\frac{82}{3}$ (5)

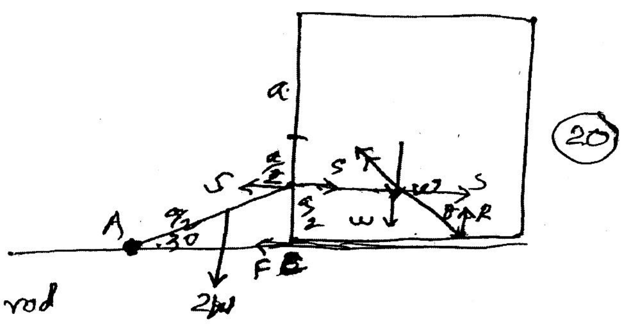
distance is $\frac{82}{3}$ cm in extended DA. from A.

Eqn of the resultant is

$y = 1x - \frac{82}{3}$

$3y + 3x + 82 = 0$ (5)

(15)



(20)

for rod

$A\circlearrowleft \quad \frac{a}{2}.S = 2W.\frac{a}{2}\cos 30$ (10)

$S = W\sqrt{3}$

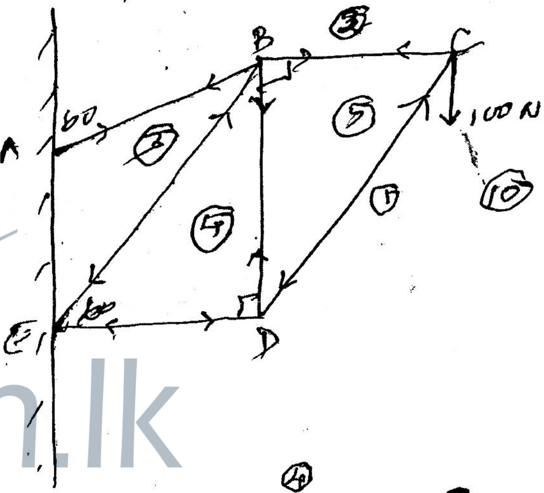
for Cube $\uparrow\ R = W$ (10)

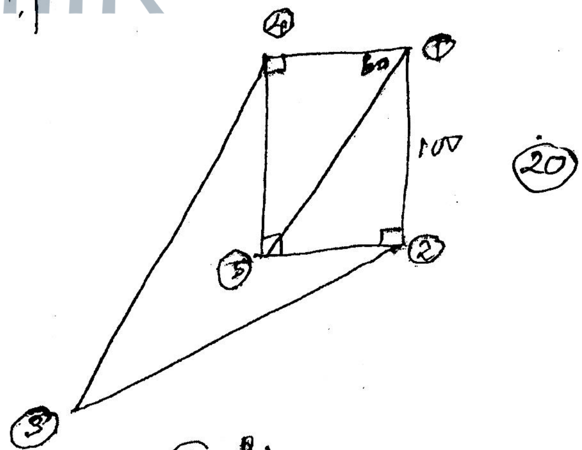
$\leftarrow\ F = S = W\sqrt{3}$ (10)

$\frac{F}{R} \le \mu$ (10)

$\sqrt{3} \le \mu$.

b)

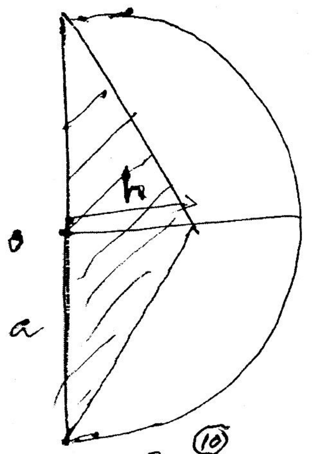




Rod	Thrust	Tension
AB	—	200 N
BC	—	$100/\sqrt{3}$
CD	$200/\sqrt{3}$	—
DB	—	100
DE	$100/\sqrt{3}$	—
BE	$400/\sqrt{3}$	—

(60)

16) Theory (50)



$\frac{1}{3}\pi a^2 \rho g(2a-h)\bar{x} = \frac{2}{3}\pi a^3\rho g\frac{3}{8}a - \frac{1}{3}\pi a^2 h\rho g\frac{h}{4}$ (10) (10) (10)

$\bar{x} = \frac{3a^2-h^2}{4(2a-h)}$ (10)

$\bar{x} = h$ (10)

$\Rightarrow \frac{3a^2-h^2}{4(2a-h)} = h$ (10)

$3a^2-h^2 = 8ah-4h^2$

$3h^2-8ah+3a^2=0$ (10)

$h = \frac{8a\pm\sqrt{64a^2-36a^2}}{6}$ (10)

$= \frac{8a\pm 2\sqrt{7}a}{6}$

$= \left(\frac{4\pm\sqrt{7}}{3}\right)a$ (10)

$= \left(\frac{4-\sqrt{7}}{3}\right)a \quad (\because h<a)$ (10)

17 a) i $A\cup B = (A\cap B')\cup B$ (5)

$(A\cap B')\cap B = \phi$ (5)

$\Rightarrow P[(A\cap B')\cup B] = P(A\cap B') + P(B)$ (10)

$P(A\cup B) = P(A) - P(A\cap B) + P(B)$ (10)

$= P(A) + P(B) - P(A\cap B)$

ii $B = A' \Rightarrow P(A\cup A') = P(A) + P(A') - P(A\cap A')$ (10)

$P(S) = P(A) + P(A') - P(\phi)$

$1 = P(A) + P(A')$ (5)

$P(A/B)$ - explaination (10)

$P(A/B) = \frac{P(A\cap B)}{P(B)}$ (5)

$P(A) = \frac{2}{5},\ P(B) = \frac{1}{2},\ P(A\cup B) = \frac{4}{5}$

iii $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{4}{5} = \frac{2}{5} + \frac{1}{2} - P(A\cap B)$

$P(A\cap B) = \frac{1}{10}$ (10)

$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{1/10}{1/2} = \frac{1}{5}$ (10)

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{1/10}{2/5} = \frac{1}{4}$ (5)

$P(B'/A') = \frac{P(A'\cap B')}{P(A')} = \frac{P(A\cup B)'}{P(A')}$

$= \frac{1-P(A\cup B)}{1-P(A)}$ (5)

$= \frac{1-4/5}{1-2/5} = \frac{1/5}{3/5} = \frac{1}{3}$ (5)

b) $P(A\cap B) = P(A)\cdot P(B)$ (5)

$P(A'\cap B) = P(B) - P(A\cap B)$ (5)

$= P(B) - P(A)\cdot P(B)$

$= P(B)[1-P(A)]$

$= P(B)\cdot P(A')$ (5)

$\Rightarrow B, A'$ are independent

$P(A'\cap B') = P(A\cup B)'$

$= 1 - P(A\cup B)$ (5)

$= 1 - [P(A)+P(B)-P(A\cap B)]$

$= 1 - P(A) - P(B)[1-P(A)]$

$= [1-P(A)][1-P(B)]$ (5)

$= P(A')\cdot P(B')$ (5)

$\Rightarrow A', B'$ are independant

c) i $\frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$ (5)

ii $\frac{2}{3}\cdot\frac{3}{4} = \frac{1}{2}$ (5)

iii $\frac{1}{4}+\frac{1}{3}-\frac{1}{12} = \frac{1}{2}$ (5)

iv $\frac{1}{4}+\frac{1}{3}-\frac{1}{6} = \frac{1}{4}$ (5)