வடமாகாணக் கல்வித் திணைக்களத்துடன் இணைந்து தொண்டைமானாறு
வெளிக்கள நிலையம் நடாத்தும் தவணைப் பரீட்சை, மார்ச் - 2020

Conducted by Field Work Centre, Thondaimanaru
In Collaboration with Provincial Department of Education Northern Province
Term Examination, March - 2020

FWC

Grade - 12 (2021) | **Combined Maths** | **Marking Scheme**

1. $y = x^2 - \lambda x + 3\lambda - 5$

$\Delta < 0$ (5)

$\lambda^2 - 4(1)(3\lambda - 5) < 0$ (5)

$\lambda^2 - 12\lambda + 20 < 0$ (5)

$(\lambda - 10)(\lambda - 2) < 0$ (5)

$2 < \lambda < 10$ (5)

$\boxed{25}$

2. $\dfrac{2x-4}{x-3} \le 1$

$\Rightarrow \dfrac{(2x-4)-(x-3)}{x-3} \le 0$ (5)

$\Rightarrow \dfrac{x-1}{x-3} \le 0$ (5)

	$x<1$	$1<x<3$	$x>3$
$x-1$	$(-)$	$(+)$	$(+)$
$x-3$	$(-)$	$(-)$	$(+)$
$\frac{x-1}{x-3}$	$(+)$	$(-)$	$(+)$

(10)

$1 \le x < 3$ (5)

$\boxed{25}$

3. $\dfrac{8}{(x^2-1)(x+3)}$

$= \dfrac{8}{(x-1)(x+1)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$ (5) (5)

$8 \equiv A(x+1)(x+3) + B(x-1)(x+3) + C(x^2-1)$

$x = 1 \Rightarrow 8A = 8 \Rightarrow A = 1$
$x = -1 \Rightarrow -4B = 8 \Rightarrow B = -2$
$x = -3 \Rightarrow 8C = 8 \Rightarrow C = 1$ (10)

$\dfrac{8}{(x^2-1)(x+3)} = \dfrac{1}{x+3} - \dfrac{2}{x+1} + \dfrac{1}{x-1}$ (5)

$\boxed{25}$

4. $a\cos A = b\cos B$ — cosine rule (5)

$\Rightarrow a\left(\dfrac{b^2+c^2-a^2}{2bc}\right) = b\left(\dfrac{c^2+a^2-b^2}{2ca}\right)$ (5)

$\Rightarrow a^2(b^2+c^2-a^2) = b^2(a^2+c^2-b^2)$

$\Rightarrow a^2(b^2+c^2-a^2) - b^2(a^2+c^2-b^2) = 0$

$\Rightarrow a^2c^2 - a^4 - b^2c^2 + b^4 = 0$ (5)

$\Rightarrow c^2(a^2-b^2) - (a^2-b^2)(a^2+b^2) = 0$

$\Rightarrow (a^2-b^2)(c^2-a^2-b^2) = 0$ (5)

$\Rightarrow a^2 - b^2 = 0$ or $a^2+b^2-c^2 = 0$

$\Rightarrow a = b$ or $a^2+b^2 = c^2$ (5)

$\boxed{25}$

5. $\cos^2 A = 1 - \sin^2 A$

$= 1 - \dfrac{9}{13}$

$= \dfrac{4}{13}$

$\cos A = -\dfrac{2}{\sqrt{13}}$ $\left[\because \frac{\pi}{2} < A < \pi\right]$ (5)

$\sin^2 B = 1 - \cos^2 B$

$= 1 - \dfrac{25}{26}$

$= \dfrac{1}{26}$

$\sin B = \dfrac{1}{\sqrt{26}}$ $\left[\because 0 < B < \frac{\pi}{2}\right]$ (5)

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ (5)

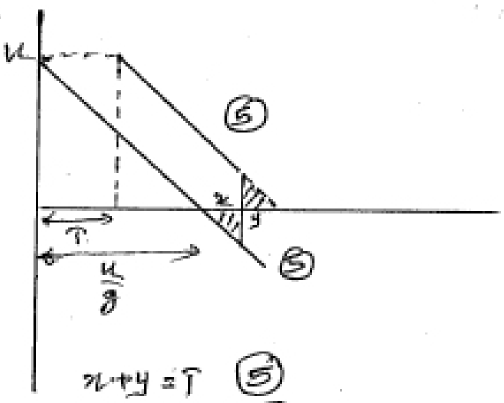
$= \dfrac{3}{\sqrt{13}}\dfrac{5}{\sqrt{26}} + \dfrac{-2}{\sqrt{13}}\dfrac{1}{\sqrt{26}}$

$\sin(A+B) = \frac{1}{\sqrt{2}}$ (5)

$A+B = \frac{3\pi}{4}$ $\left[\because \frac{\pi}{2} < A+B < \frac{3\pi}{2}\right]$ (5)

$\boxed{25}$

(6)

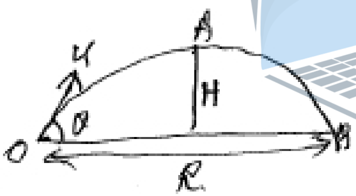


$x+y = T$ (5)

$x = y = \frac{T}{2}$ (5)

$\therefore$ Time taken to meet is $\frac{u}{g} + x = \frac{u}{g} + \frac{T}{2}$. (5) $\boxed{25}$

(7)



$\uparrow O \to A$

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$ (5)

$H = \frac{u^2\sin^2\theta}{2g}$

$O \to B \uparrow$ $s = ut + \frac{1}{2}at^2$

$0 = u\sin\theta - \frac{1}{2}gt$ (5)

$t = \frac{2u\sin\theta}{g}$

$\rightarrow$ $R = u\cos\theta . t$ (5)

$= \frac{2u^2\sin\theta\cos\theta}{g}$

$R = 5H$

$\Rightarrow \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin^2\theta}{2g}$ (5)

$\tan\theta = 4$.

$\theta = \tan^{-1}(4)$ (5)

$\boxed{25}$

(8) $|\underline{a}+\underline{b}+\underline{c}|^2 = (\underline{a}+\underline{b}+\underline{c}).(\underline{a}+\underline{b}+\underline{c})$

(5) $= a^2 + b^2 + c^2 + 2(\underline{a}.\underline{b} + \underline{b}.\underline{c} + \underline{c}.\underline{a})$

(5) $= 3\lambda^2$ $(\because |\underline{a}| = |\underline{b}| = |\underline{c}| = \lambda)$

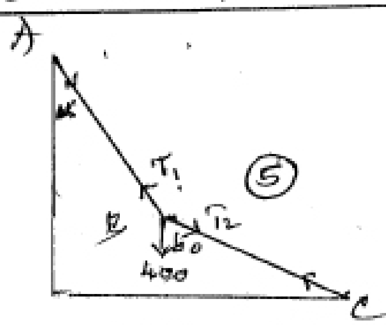
$(\underline{a}+\underline{b}+\underline{c}).\underline{a} = a^2 + \underline{a}.\underline{b} + \underline{a}.\underline{c}$ (5)

$|\underline{a}+\underline{b}+\underline{c}|.|\underline{a}|\cos\theta = \lambda^2$ (5)

$\cos\theta = \frac{1}{\sqrt{3}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (5) $\boxed{25}$

(9)



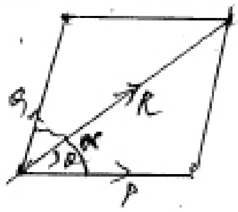
$\frac{T_1}{\sin 60} = \frac{T_2}{\sin(\pi-\alpha)} = \frac{400}{\sin(120+\alpha)}$ (10)

$\frac{T_1}{\frac{\sqrt{3}}{2}} = \frac{T_2}{\sin 30} = \frac{400}{\sin 150}$

$T_1 = 400\sqrt{3}$ N (5) $\boxed{25}$

$T_2 = 400$ N (5)

(10) $\tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha}$ — (1) (5)



If Q is reversed

$\tan(90-\theta) = \frac{Q\sin\alpha}{P-Q\cos\alpha}$ — (2) (5)

(1).(2) $\Rightarrow$
$\tan\theta.\cot\theta = \frac{Q^2\sin^2\alpha}{P^2-Q^2\cos^2\alpha}$ (5)

$P^2 - Q^2\cos^2\alpha = Q^2\sin^2\alpha$ (5)

$P^2 = Q^2$

$P = Q$ (5)

$\boxed{25}$

11.

(a) (i) $f(x) = 0$

$\Rightarrow x^2 - (a-3)x + a = 0$

$\alpha + \beta = a - 3$ (5), $\alpha\beta = a$ (5)

$(\alpha-2) + (\beta-2)$

$= (\alpha+\beta) - 4$ (5)

$= a - 7$ (5)

$(\alpha-2)(\beta-2)$

$= \alpha\beta - 2(\alpha+\beta) + 4$ (5)

$= -(a-10)$ (5)

The required equation is

$(x - (\alpha-2))(x - (\beta-2)) = 0$

$x^2 - (a-7)x - (a-10) = 0$ (5)

$\boxed{35}$

(ii) The discriminant $= (a-3)^2 - 4a$ (10)

Since $f(x)=0$ has real roots,

we have $\Delta \geq 0$ (5)

$(a-3)^2 - 4a \geq 0$

$a^2 - 10a + 9 \geq 0$

$(a-1)(a-9) \geq 0$ (5)

$a \leq 1$ or $a \geq 9$ (5)

$\boxed{25}$

Since both α and β are greater than 2

$\Delta \geq 0$, $(\alpha-2)+(\beta-2) > 0$ and $(\alpha-2)(\beta-2) > 0$ (5)

$\Delta \geq 0 \Rightarrow a \leq 1$ or $a \geq 9$ —— ①

$(\alpha-2)+(\beta-2) > 0 \Rightarrow a - 7 > 0$

$\Rightarrow a > 7$ —— ② (5)

$(\alpha-2)(\beta-2) > 0 \Rightarrow -(a-10) > 0$

$\Rightarrow a < 10$ —— ③ (5)

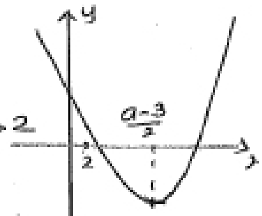
①, ②, ③ $\Rightarrow$

$9 \leq a < 10$ (5)

$\boxed{20}$

Aliter

Since $\alpha > 2$ and $\beta > 2$



$\Delta \geq 0$, $f(2) > 0$ and $\frac{a-3}{2} > 2$ (5)

$f(2) > 0 \Rightarrow 4 - 2(a-3) + a > 0$

$a < 10$ —— ① (5)

$\frac{a-3}{2} > 2 \Rightarrow a > 7$ —— ② (5)

$\Delta \geq 0 \Rightarrow a \leq 1$ or $a \geq 9$ —— ③

①, ②, ③ $\Rightarrow 9 \leq a < 10$ (5)

$\boxed{20}$

(b) $P(x) = 4x^3 + 5x^2 + \lambda x + \mu$

$P(1) = 8$ (5) $\Rightarrow 4 + 5 + \lambda + \mu = 8$ (5)

$\lambda + \mu = -1$ —— ①

$P(-1) = -2$ (5) $\Rightarrow -4 + 5 - \lambda + \mu = -2$ (5)

$\lambda - \mu = 3$ —— ②

$\lambda = 1$ (5)

$\mu = -2$ (5)

$\boxed{30}$

$P(x) = 4x^3 + 5x^2 + x - 2$

$4x^3 + 5x^2 + x - 2 \equiv (x^2 + x - 1)(Ax + B) + Cx + D$ (10)

x^3: $4 = A$ (5)

x^2: $5 = B + A$ (5)

x: $1 = B - A + C$ (5)

x^0: $-2 = -B + D$ (5)

$A = 4$, $B = 1$, $C = 4$, $D = -1$

Quotient $= 4x + 1$ (5), Remainder $= 4x - 1$ (5)

$\boxed{40}$

Aliter

$$\begin{array}{r}
4x+1 \\
x^2+x-1 \overline{\big) 4x^3+5x^2+x-2} \\
\underline{4x^3+4x^2-4x} \\
x^2+5x-2 \\
\underline{x^2+x-1} \\
4x-1
\end{array}$$

(30)

Quotient $= 4x + 1$

Remainder $= 4x - 1$ (5)

$\boxed{40}$

12 a) $a, b, c \in \mathbb{R}^+$

let $\log_a b = x \Leftrightarrow a^x = b$ (5) —(1)

Now consider $\log_c b = \log_c a^x$ (from (1)) (5)

$\log_c b = x \log_c a$ (5)

$x = \dfrac{\log_c b}{\log_c a} = \log_a b$

Put $c = b$ $\quad \log_a b = \dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a}$ (5) (5)

$\boxed{25}$

1. $\log_2\left(x+\frac{15}{x}\right) + 3\log_{\left(x+\frac{15}{x}\right)} 16 = 7$

$\log_2\left(x+\frac{15}{x}\right) + 3\log_{\left(x+\frac{15}{x}\right)} 2^4 = 7$

$\log_2\left(x+\frac{15}{x}\right) + \dfrac{12}{\log_2\left(x+\frac{15}{x}\right)} = 7$ (5)

Put $\log_2\left(x+\frac{15}{x}\right) = t$

then $t + \dfrac{12}{t} = 7$ (5)

$\Rightarrow t^2 - 7t + 12 = 0 \Rightarrow t = 3$ or 4 (5)

$t = 3$

$\log_2\left(x+\frac{15}{x}\right) = 3$

$x + \frac{15}{x} = 8$ (5)

$x^2 - 8x + 15 = 0$

$x = 3$ or 5 (5)

$t = 4$

$\log_2\left(x+\frac{15}{x}\right) = 4$

$x + \frac{15}{x} = 16$ (5)

$x^2 - 16x + 15 = 0$

$x = 1$ or 15

$\boxed{30}$

2. $\log_{27} \frac{x}{y} = \frac{2}{3}$

$\dfrac{1}{\log_{\frac{x}{y}} 3^3} = \dfrac{2}{3}$

$\dfrac{1}{3}\log_3 \dfrac{x}{y} = \dfrac{2}{3}$ (5)

$\log_3 x - \log_3 y = 2$ —(1) (5)

$\log_{27} x \cdot \log_3 y = 5$

$\frac{1}{3}\log_3 x \cdot \log_3 y = 5$

$\log_3 y = \dfrac{15}{\log_3 x}$ (5) —(2)

(1) & (2)

$\log_3 x - \dfrac{15}{\log_3 x} = 2$ (5)

take

$\log_3 x = t \Rightarrow x = 3^t$

$\Rightarrow t - \frac{15}{t} = 2$

$t^2 - 2t - 15 = 0 \Rightarrow t = 5$ or -3 (5)

$x = 3^5$ or 3^{-3} (5)

$\log_3 y = \log_3 x - 2$

$t = 5$

$\log_3 y = 5 - 2 = 3$

$y = 3^3$ (5)

$t = -3$

$\log_3 y = -3 - 2$

$y = 3^{-5}$

$\boxed{35}$

b) $a, b, c \in \mathbb{R}$

1. $(a-b)^2 \geq 0$ (5)

$a^2 + b^2 - 2ab \geq 0$ (5)

$a^2 + b^2 \geq 2ab$ (5)

2. $(a^2+b^2)(a^4+b^4) - (a^3+b^3)^2$

$= (a^6 + a^2b^4 + b^2a^4 + b^6) - (a^6 + 2a^3b^3 + b^6)$ (5)

$= a^2b^2(a^2 - 2ab + b^2)$ (5)

$= a^2b^2(a-b)^2 \geq 0$ (5) (5)

$\Rightarrow (a^2+b^2)(a^4+b^4) \geq (a^3+b^3)^2$ (5)

3). $(a-b)^2 \geq 0$ (5)

$(a-b)^2 + (a+b)^2 \geq (a+b)^2$ (5)

$a^2 - 2ab + b^2 + a^2 + 2ab + b^2 \geq (a+b)^2$ (5)

$2(a^2+b^2) \geq (a+b)^2$ (5)

$\boxed{60}$

13. (a)

(i) $\lim_{n\to\infty}\left\{\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n}{n^2}\right\}$

$= \lim_{n\to\infty}\left\{\frac{1+2+\cdots+n}{n^2}\right\}$ (5)

$= \lim_{n\to\infty}\frac{\frac{n}{2}(n+1)}{n^2}$ (10)

$= \lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)$ (5)

$= \frac{1}{2}(1+0)$ (5)

$= \frac{1}{2}$ (5) $\boxed{30}$

(ii) $\lim_{x\to 0}\frac{3x-\sin 2x}{3\sin x+4x}$

$= \lim_{x\to 0}\frac{3-\frac{\sin 2x}{x}}{3\frac{\sin x}{x}+4}$ (10)

$= \frac{\lim_{x\to 0}3-2\lim_{2x\to 0}\frac{\sin 2x}{2x}}{3\lim_{x\to 0}\frac{\sin x}{x}+\lim_{x\to 0}4}$ (10)

$= \frac{3-2(1)}{3(1)+4}$ (5)

$= \frac{1}{7}$ (5) $\boxed{30}$

(b)(i) $x=\sin t$

$\frac{dx}{dt}=\cos t$ (5)

$y=t\cos^2 t$

$\frac{dy}{dt}=t\,2\cos t(-\sin t)+\cos^2 t$ (10)

$= \cos t(\cos t-2t\sin t)$

$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ (5)

$= \cos t(\cos t-2t\sin t)\times\frac{1}{\cos t}$ (5)

$= \cos t-2t\sin t$

$\boxed{25}$

(ii) $\frac{d^2y}{dx^2}=\left[-\sin t-(2t\cos t+\sin t\,(2))\right]\frac{dt}{dx}$ (10)

$= -(3\sin t+2t\cos t)\frac{1}{\cos t}$ (5)

$= -(3\tan t+2t)$ $\boxed{15}$

(c) $\cos(y-\alpha)=x\cos y$

differentiate w.r.t. x, we get

$-\sin(y-\alpha)\frac{dy}{dx}=x(-\sin y)\frac{dy}{dx}+\cos y$ (10)

$\left[x\sin y-\sin(y-\alpha)\right]\frac{dy}{dx}=\cos y$ (5)

$\left[\frac{\cos(y-\alpha)}{\cos y}\sin y-\sin(y-\alpha)\right]\frac{dy}{dx}=\cos y$ (5)

$\frac{\sin y\cos(y-\alpha)-\cos y\sin(y-\alpha)}{\cos y}\frac{dy}{dx}=\cos y$ (5)

$\frac{\sin(y-(y-\alpha))}{\cos y}\frac{dy}{dx}=\cos y$ (5)

$\frac{dy}{dx}=\frac{\cos^2 y}{\sin\alpha}$ (5)

$\boxed{35}$

$\frac{d^2y}{dx^2}=\frac{1}{\sin\alpha}\,2\cos y(-\sin y)\frac{dy}{dx}$ (10)

$= -\frac{2\sin y\cos y}{\sin\alpha}\times\frac{\cos^2 y}{\sin\alpha}$ (5)

$= -\frac{2\sin y\cos^3 y}{\sin^2\alpha}$ $\boxed{15}$

Q14]

a)

(i) $\sin 7\theta + \sin\theta = \sin 4\theta$

$2\sin 4\theta \cos 3\theta - \sin 4\theta = 0$ (5)

$\sin 4\theta (2\cos 3\theta - 1) = 0$ (5)

$\sin 4\theta = 0$ or $\cos 3\theta = \frac{1}{2}$

$\sin 4\theta = \sin 0$ (5) $\quad \cos 3\theta = \cos\frac{\pi}{3}$ (5)

$4\theta = n_1\pi \quad 3\theta = 2n_2\pi \pm \frac{\pi}{3}$

$\theta = \frac{n_1\pi}{4}$ (5) $\quad \theta = \frac{2n_2\pi}{3} \pm \frac{\pi}{9}$ (5)

$n_1, n_2 \in \mathbb{Z}$

$\boxed{30}$

(ii) $\cos\theta + \cos 2\theta + \cos 3\theta = \sin\theta + \sin 2\theta$

$2\cos 2\theta \cos\theta + \cos 2\theta = \sin\theta + 2\sin\theta\cos\theta$ (10)

$\cos 2\theta (2\cos\theta + 1) = \sin\theta (2\cos\theta + 1)$

$(2\cos\theta + 1)(\cos 2\theta - \sin\theta) = 0$ (5)

$2\cos\theta + 1 = 0$ or $\cos 2\theta - \sin\theta = 0$

$\cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$ (5) $\quad 2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\theta = 2n_1\pi \pm \frac{2\pi}{3}$ (5) $\quad \sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\sin\theta = \sin\frac{\pi}{6} \quad \theta = n_3\pi + (-1)^{n_3}\frac{\pi}{2}$ (10)

$\theta = n_2\pi + (-1)^{n_2}\frac{\pi}{6}$

$n_1, n_2, n_3 \in \mathbb{Z}$

$\boxed{35}$

b) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k}$ (say) (*) (5)

(i) $\frac{b-c}{a} = \frac{k\sin B - k\sin C}{k\sin A}$ (5) (by *)

$= \frac{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$ (10)

$\frac{b-c}{a} = \frac{\sin\left(\frac{B-C}{2}\right)}{\cos\frac{A}{2}}$ (5) $\left(\because \frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2},\ \cos\left(\frac{B+C}{2}\right) = \sin\frac{A}{2}\right)$

$\Rightarrow a\sin\left(\frac{B-C}{2}\right) = (b-c)\cos\frac{A}{2}$

$\boxed{25}$

(ii) $2b = a + c$ (5)

$2k\sin B = k\sin A + k\sin C$ (by *) (5)

$2\left(2\sin\frac{B}{2}\cos\frac{B}{2}\right) = 2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)$ (5)

$2\sin\frac{B}{2} = \cos\left(\frac{A-C}{2}\right)$ (5) $(\because A+B+C=\pi)$

$2\cos\left(\frac{A+C}{2}\right) = \cos\left(\frac{A-C}{2}\right)$ (5)

$2\left\{\cos\frac{A}{2}\cos\frac{C}{2} - \sin\frac{A}{2}\sin\frac{C}{2}\right\}$ (5) $= \cos\frac{A}{2}\cos\frac{C}{2} + \sin\frac{A}{2}\sin\frac{C}{2}$

$\cos\frac{A}{2}\cos\frac{C}{2} = 3\sin\frac{A}{2}\sin\frac{C}{2}$ (5)

$\tan\frac{A}{2}\tan\frac{C}{2} = \frac{1}{3}$

$\boxed{35}$

c) $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ — (1)

$\alpha = \tan^{-1}(2x)$

$\beta = \tan^{-1}(3x)$ (say)

(1) $\Rightarrow \alpha + \beta = \frac{\pi}{4}$ (5)

$\tan(\alpha+\beta) = 1$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$ (5)

$\frac{2x + 3x}{1 - (2x)(3x)} = 1$ (5)

$6x^2 + 5x - 1 = 0$ (5)

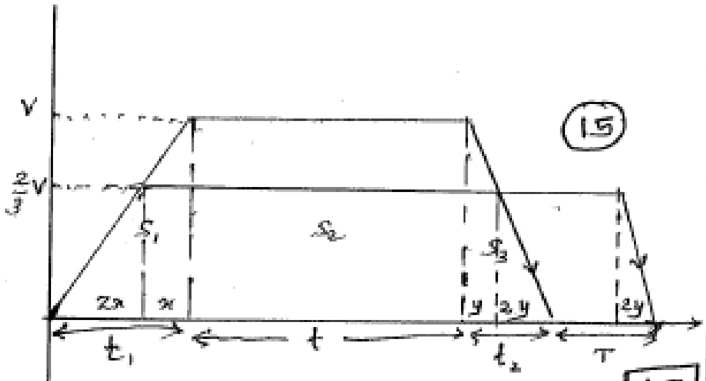
$(6x-1)(x+1) = 0$

$6x - 1 = 0 \quad (\because x > 0)$

$x = \frac{1}{6}$ (5)

$\boxed{25}$

(15) (i)



(15)

$\boxed{15}$

I $\quad S_1 = \frac{1}{2}vt_1$ (5) $\qquad S_2 = vt$ (5) $\qquad S_3 = \frac{1}{2}vt_2$ (5)

$$\frac{3}{4}v = \frac{S_1+S_2+S_3}{t_1+t_2+t}$$ (5)

$$= \frac{\frac{1}{2}v(t_1+t_2+2t)}{t_1+t_2+t}$$

$\Rightarrow t_1+t_2 = t$ (5)

$\boxed{25}$

II $\quad S = S_1+S_2+S_3$

$= \frac{v}{2}(t_1+t_2+2t)$ (5)

$= \frac{3}{2}vt$ (5)

$x = \frac{t_1}{3}$ (5), $\quad y = \frac{t_2}{3}$ (5)

$S = \frac{1}{2}\left[t_1+t_2+t+T+x+t+y+T\right]\frac{2}{3}v$ (5)

$= \frac{v}{3}\left[\frac{10}{3}t+2T\right] = \frac{3}{2}vt$ (5)

$T = \frac{7t}{12}$ (5)

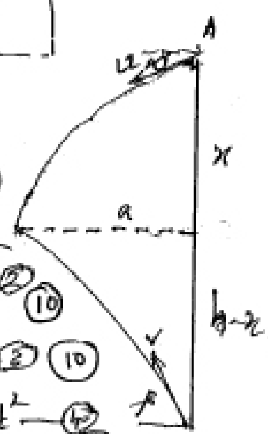
Total time $= t_1+t_2+t+T$

$= 2t+\frac{7t}{12}$ (5)

$= \frac{31t}{12}$ (5)

$\boxed{45}$

b)



$A \to C$

$\leftarrow \quad u\cos\alpha \cdot t = a$ ——— ① (10)

$\downarrow \quad x = u\sin\alpha \cdot t + \frac{1}{2}gt^2$ ——— ② (10)

$B \to C$

$\leftarrow \quad v\cos\beta \, t = a$ ——— ③ (10)

$\uparrow \quad h-x = v\sin\beta t - \frac{1}{2}gt^2$ ——— ④ (10)

① + ③ $\quad u\cos\alpha = v\cos\beta$ (5)

② + ④ $\quad h = (u\sin\alpha + v\sin\beta)t$ (5)

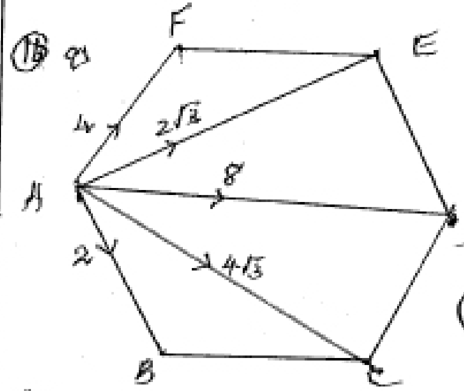
$= u\sin\alpha \cdot t + v\sin\beta \cdot t$

$= u\sin\alpha \cdot \frac{a}{u\cos\alpha} + v\sin\beta \cdot \frac{a}{v\cos\beta}$ (10)

$= a(\tan\alpha + \tan\beta)$ (5)

$\boxed{65}$

(16) a)



(15)

$\overrightarrow{AD}$ $\quad x = 8 + 4\sqrt{3}\cos 30 + 2\cos 60 + 2\sqrt{3}\cos 30 + 4\cos 60$ (20)

$= 20$ N (10)

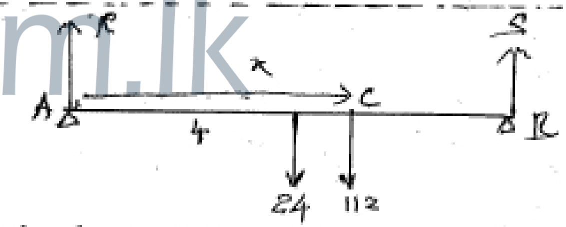
$\uparrow y \quad = 4\sin 60 + 2\sqrt{3}\sin 30 - 4\sqrt{3}\sin 30 - 2\sin 60$ (20)

$= 0$ (10)

$\boxed{85}$

Resultant 20 N acts along $\overrightarrow{AD}$ (10)

b)



$\uparrow \quad R+S = 24+112$ (10)

$= 136$

$\frac{R}{S} = \frac{8}{9} \Rightarrow 9R = 8S$ (10)

$R + \frac{9}{8}R = 136$ (5)

$R = 64$ (10), $\quad S = 72$ (10)

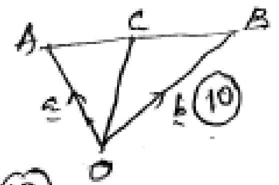
A↺ $\quad S.8 - 24.4 - 112.x = 0$ (10)

$x = \frac{30}{7}$ (10)

$AC = \frac{30}{7}$ m

$\boxed{65}$

(17) I



$\vec{OC} = \vec{OA} + \vec{AC}$ (10)

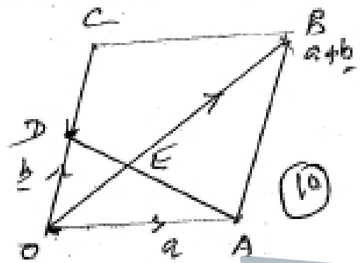
$\underline{c} = \underline{a} + \alpha\vec{AB}$ (10)

$= \underline{a} + \alpha(\underline{b} - \underline{a})$ (10)

$= (1-\alpha)\underline{a} + \alpha\underline{b}$

let $1-\alpha = \lambda$

$= \lambda\underline{a} + (1-\lambda)\underline{b}$ (10) $\boxed{50}$



$\vec{OC} = \underline{b}$

$\underline{d} = \frac{1}{2}\underline{b}$

$\vec{OE} = \underline{e} = \mu\underline{a} + (1-\mu)\underline{d}$ (10)

$= \mu\underline{a} + (1-\mu)\frac{1}{2}\underline{b}$ (10)

$\vec{OE} = \cancel{\mu}\,\alpha\vec{OB}$ (10)

$\mu\underline{a} + (1-\mu)\frac{1}{2}\underline{b} = \alpha(\underline{a}+\underline{b})$ (10)

$(\mu-\alpha)\underline{a} + [\frac{1}{2}(1-\mu) - \alpha]\underline{b} = 0$

$\Rightarrow \mu - \alpha = 0$ & $\frac{1-\mu}{2} - \alpha = 0$ (10)

(10) $\mu = \alpha$. & $1-\mu-2\mu = 0$

$\mu = \frac{1}{3} = \alpha$ (10)

$\therefore \underline{e} = \frac{1}{3}(\underline{a}+\underline{b})$ (10)

$OE : EB = 1 : 2$ (10) $\boxed{100}$