



FWC

Grade - 12 (2020)

G.C.E A/L Examination July - 2019

Fied Work Centre

Combined Mathematics

Marking Scheme

$$\begin{aligned}
 1. \quad f(x) &= a^2(x^2+1) + 2abx - 2b(a-b) \\
 &= a^2x^2 + 2abx + b^2 + a^2 - 2ab + b^2 \quad (5) \\
 &= (ax+b)^2 + (a-b)^2 \geq 0 \quad (5) \\
 &\quad (5) \quad \left[\begin{array}{l} \because (a-b)^2 \geq 0, \\ (ax+b)^2 \geq 0 \end{array} \right]
 \end{aligned}$$

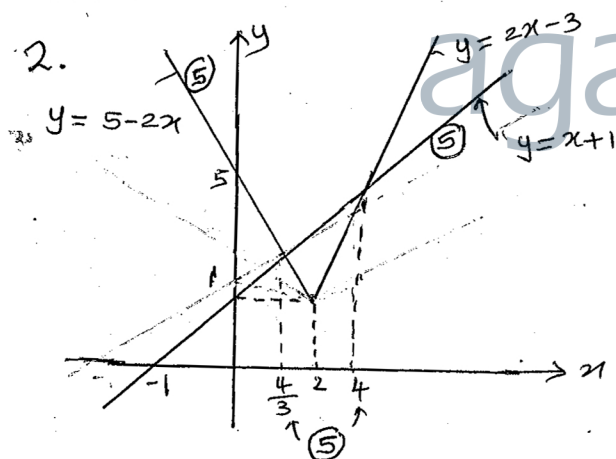
If $|a-b| = 2$, then $(a-b)^2 = 4$

$$\begin{aligned}
 f(x) &= (ax+b)^2 + 4 \quad (5) \\
 f(x)_{\min} &= 4 \quad \left[\because (ax+b)^2 \geq 0 \right] \quad (5)
 \end{aligned}$$

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$$\begin{aligned}
 3. \quad \lim_{x \rightarrow 0} \frac{\sin 3x}{(8+x)^{\frac{1}{3}} - 2} \\
 = \lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{x}}{\frac{(8+x)^{\frac{1}{3}} - 2}{x}} \quad (5) \\
 = \frac{3 \lim_{3x \rightarrow 0} \frac{\sin 3x}{3x}}{\lim_{(8+x) \rightarrow 8} \frac{(8+x)^{\frac{1}{3}} - 8^{\frac{1}{3}}}{(8+x) - 8}} \quad (5) \\
 = \frac{3 \times 1}{\frac{1}{3} 8^{-\frac{2}{3}}} \quad (5) \\
 = 36 \quad (5)
 \end{aligned}$$

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$$2|x-2| \leq x \quad (5)$$

$$2|x-2| + 1 \leq x + 1 \quad (5)$$

$$\frac{4}{3} \leq x \leq 4 \quad (5)$$

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$$\begin{aligned}
 4. \quad x &= 2 \cos \theta - \cos 2\theta \\
 \frac{dx}{d\theta} &= -2 \sin \theta + 2 \sin 2\theta \quad (5) \\
 &= 2(2) \cos \frac{3\theta}{2} \sin \frac{\theta}{2} \\
 y &= 2 \sin \theta - \sin 2\theta \\
 \frac{dy}{d\theta} &= 2 \cos \theta - 2 \cos 2\theta \quad (5) \\
 &= 2(2) \sin \frac{3\theta}{2} \sin \frac{\theta}{2} \\
 \frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{4 \sin \frac{3\theta}{2} \sin \frac{\theta}{2}}{4 \cos \frac{3\theta}{2} \sin \frac{\theta}{2}} \quad (5) \\
 &= \tan \frac{3\theta}{2}
 \end{aligned}$$

$$\frac{dy}{dx} = 1 \Rightarrow \tan \frac{3\theta}{2} = 1 \Rightarrow \theta = \frac{\pi}{6} \quad (5)$$

$$P \equiv \left(\frac{2\sqrt{3}-1}{2}, \frac{2-\sqrt{3}}{2} \right) \quad (5)$$

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⑥ $t=0$ $f=3$
 $b=2$ x M
 $v=16$

$A \rightarrow O \rightarrow M$ $x = \frac{1}{2} \times 3 t^2$ (5)
 $B \rightarrow$ $x = 16(t-2)$ (5)

$3t^2 = 32t + b4 = 0$
 There are two values for t

$t_1 + t_2 = \frac{32}{3}$, $t_1 t_2 = \frac{b4}{3}$ (10)

$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1 t_2$

$= \left(\frac{32}{3}\right)^2 - 4 \times \frac{b4}{3}$

$= \frac{256}{9}$

$|t_1 - t_2| = \frac{16}{3}$

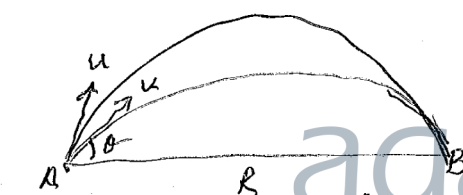
distance between the points

$= 16(t_1 - t_2)$

$= 16 \times \frac{16}{3}$

$= \frac{256}{3}$ (5)

⑦



$A \rightarrow B$ $s = ut + \frac{1}{2}at^2$

$0 = u \sin \theta - \frac{1}{2}gt^2$

$t = \frac{2u \sin \theta}{g}$ (5)

$\rightarrow R = u \cos \theta \times \frac{2u \sin \theta}{g}$

$= \frac{u^2 \sin 2\theta}{g}$

$= \frac{u^2}{g} \sin 2\left(\frac{\pi}{2} - \theta\right)$ (5)

Therefore angles of projection are

$\theta, \frac{\pi}{2} - \theta$
 difference between the directions

$\left(\frac{\pi}{2} - 2\theta\right)$ (5)

⑧ $|a \cdot b| = |a||b| \cos \theta$ (5)

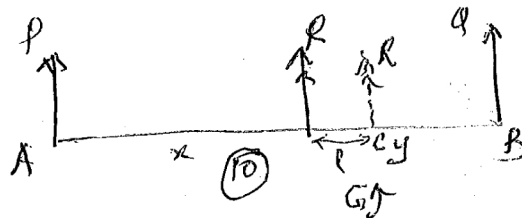
$|a \times b| = |a||b| \sin \theta$ (5)

$\frac{|a \cdot b|}{|a \times b|} = \frac{1 \cos \theta}{\sin \theta}$ (5)

$\Rightarrow \tan \theta = 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ (10)

⑨



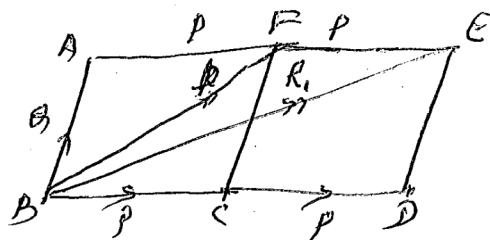
$\frac{P}{Q} = \frac{y}{x}$ (5)

⑩ $Q(y-l) + G - P(x+l) = 0$ (5)

$Qy - Ql + G - Px - Pl = 0$

$l = \frac{G}{P+Q}$ (5)

⑩



$AF = FE = BF$ (5)

$\Rightarrow \angle ABE = \frac{\pi}{2}$

$R_1^2 + Q^2 = (2P)^2$

$R_1^2 = 4P^2 - Q^2$

$R_1 = \sqrt{4P^2 - Q^2}$

$$11. ax^2 - 2bx - (a-b) = 0$$

The discriminant

$$\Delta = 4b^2 + 4a(a-b) \quad (10)$$

$$= 4b^2 + 4a^2 - 4ab$$

$$= (2a-b)^2 + 3b^2 \geq 0 \quad (5) \quad (5)$$

Hence The roots are real. (5)

[25]

$$\alpha + \beta = \frac{2b}{a} \quad (5) \quad \alpha\beta = \frac{(b-a)}{a} \quad (5)$$

$$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 4\alpha\beta \quad (5)$$

$$= \frac{4b^2}{a^2} - 4\frac{(b-a)}{a} \quad (5)$$

$$= \frac{4b^2 + 4a^2 - 4ab}{a^2}$$

$$(\alpha - \beta) = \pm \frac{\sqrt{4b^2 + 4a^2 - 4ab}}{a} \quad (5)$$

$\alpha > \beta$

$$(\alpha - \beta) = \frac{2\sqrt{b^2 + a^2 - ab}}{a} \quad (5)$$

$$\text{IF } (\alpha - \beta) = 2$$

$$\frac{2\sqrt{b^2 + a^2 - ab}}{a} = 2 \quad (5)$$

$$b^2 + a^2 - ab = a^2$$

$$b(b-a) = 0 \quad (5)$$

$$b \neq 0 \Rightarrow a = b \quad (5)$$

[45]

$$\text{III } (\alpha - \beta) + (\alpha + \beta) = \frac{2\sqrt{a^2 - ab + b^2}}{a} + \frac{2b}{a} \quad (5)$$

$$= \frac{2}{a} (\sqrt{a^2 - ab + b^2} + b) \quad (5)$$

$$(\alpha - \beta) \cdot (\alpha + \beta) = \frac{4b\sqrt{a^2 - ab + b^2}}{a^2} \quad (5)$$

The required equation is

$$x^2 - \frac{2}{a} (\sqrt{a^2 - ab + b^2} + b) x +$$

$$\frac{4b}{a^2} \sqrt{a^2 - ab + b^2} = 0 \quad (10)$$

$$a^2 x^2 - 2(\sqrt{a^2 - ab + b^2} + b)x + 4b\sqrt{a^2 - ab + b^2} = 0 \quad (25)$$

b) $(x+2)$ is a factor of $9cx$

$$\therefore 9cx = x^2 + px + b$$

$$9(-2) = 0 = 4 - 2p + b \quad (5)$$

$$2p = 10 \Rightarrow p = 5 \quad (5)$$

$$\therefore 9cx = x^2 + 5x + b$$

When $f(x)$ is divided by $(x-1)$
The remainder is one & Quotient is $9cx$.

$$\text{So } \frac{f(x)}{(x-1)} = 9cx + \frac{1}{(x-1)} \quad (10)$$

$$f(x) = (x^2 + 5x + b)(x-1) + 1 \quad (5)$$

$$= x^3 + 5x^2 + bx - x^2 - 5x - b + 1$$

$$= x^3 + 4x^2 + x - 5 \quad (5)$$

$$\text{Now } f(x) = (x-1)(x^2 + 5x + b) + 1 \quad (5)$$

$$f(x) = (x-1)(x+2)(x+3) + 1$$

$$f(x) = (x^2 + x - 2)(x+3) + 1 \quad (5)$$

$$\frac{f(x)}{(x^2 + x - 2)} = (x+3) + \frac{1}{(x^2 + x - 2)} \quad (5)$$

$$R = 1 \quad (5)$$

When $f(x)$ is divided by $(x^2 + x - 2)$ The remainder is One.

[03]

[55]

12.

(a) Let $f(x) = \sin 2x$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (10)$$

$$= \lim_{h \rightarrow 0} \frac{\sin 2(x+h) - \sin 2x}{h} \quad (10)$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos(2x+h) \sin h}{h} \quad (10)$$

$$= \lim_{h \rightarrow 0} 2 \cos(2x+h) \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} \quad (10)$$

$$= 2 \cos 2x \quad (10)$$

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(c) $y = \ln(\cos x)$

$$\frac{dy}{dx} = \frac{1}{\cos x} (-\sin x) \quad (10)$$

$$= -\tan x$$

$$\frac{d^2y}{dx^2} = -\sec^2 x = -(1 + \tan^2 x) \quad (10)$$

$$\frac{d^2y}{dx^2} = -\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} \quad (5)$$

$$\frac{d^3y}{dx^3} = -2 \frac{dy}{dx} \frac{d^2y}{dx^2} \quad (10)$$

$$\Rightarrow \frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} \cdot \frac{dy}{dx} = 0 \quad (5)$$

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(b) (i) $x = 2 \sin \theta$

$$\frac{dx}{d\theta} = 2 \cos \theta \quad (5)$$

$$y = 2 + \cos \theta$$

$$\frac{dy}{d\theta} = -\sin \theta \quad (5)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \quad (5)$$

$$= \frac{-\sin \theta}{2 \cos \theta} \quad (5)$$

$$= -\frac{1}{2} \tan \theta$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \sec^2 \theta \frac{d\theta}{dx} \quad (10)$$

$$= -\frac{1}{2} \sec^2 \theta \frac{1}{2 \cos \theta} \quad (5)$$

$$= -\frac{1}{4} \sec^3 \theta$$

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13.

a) Theory — 5

$$(1) \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k} \text{ (say)} \quad (5)$$

$$\frac{a}{b+c} = \frac{k \sin A}{k \sin B + k \sin C} \quad (5)$$

$$= \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{2 \sin \left(\frac{B+C}{2}\right) \cos \left(\frac{B-C}{2}\right)} \quad (10)$$

$$= \frac{\sin \frac{A}{2}}{\cos \left(\frac{B-C}{2}\right)} \quad \left(\begin{array}{l} \because \frac{B+C}{2} = \frac{\pi - A}{2} \\ \sin \left(\frac{\pi - A}{2}\right) = \cos \frac{A}{2} \end{array} \right) \quad (5)$$

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(ii) Let $u = \sin x$ and $v = \cos^2 x$.

$$\text{Then } \frac{du}{dx} = \cos x \quad (5), \quad \frac{dv}{dx} = 2 \cos x (-\sin x) \quad (10)$$

$$\frac{dv}{du} = \frac{\frac{dv}{dx}}{\frac{du}{dx}} = \frac{-2 \sin x \cos x}{\cos x} \quad (5)$$

$$= -2 \sin x \quad (5)$$

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13 ⇒

$$(ii) \frac{b-c}{b+c} = \frac{k \sin B - k \sin C}{k \sin B + k \sin C} \quad (5)$$

$$= \frac{\sin B - \sin C}{\sin B + \sin C}$$

$$= \frac{2 \cos\left(\frac{B+C}{2}\right) \sin\left(\frac{B-C}{2}\right)}{2 \sin\left(\frac{B+C}{2}\right) \cos\left(\frac{B-C}{2}\right)} \quad (10)$$

$$= \cot\left(\frac{B+C}{2}\right) \tan\left(\frac{B-C}{2}\right) \quad (5)$$

$$= \tan \frac{A}{2} \tan\left(\frac{B-C}{2}\right) \quad (2)$$

$$\left(\because \frac{B+C}{2} = \frac{\pi - A}{2}\right) \quad (5)$$

$$\cot\left(\frac{B+C}{2}\right) = \tan \frac{A}{2}$$

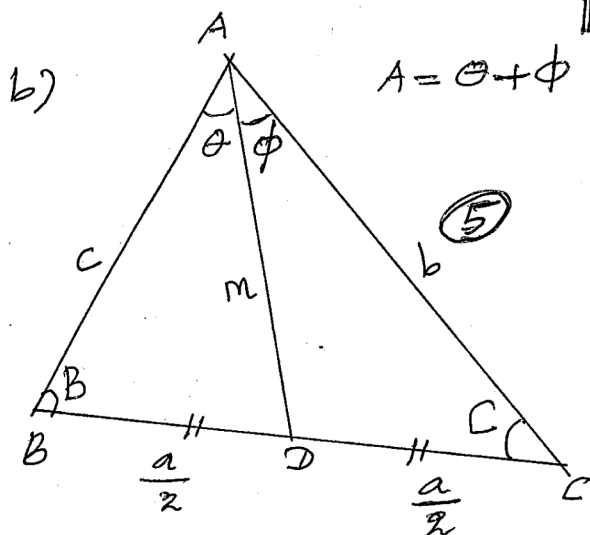
(iii)

① × ② ⇒

$$\frac{a(b-c)}{(b+c)^2} = \frac{\sin \frac{A}{2} \tan \frac{A}{2} \tan\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B-C}{2}\right)} \quad (5) \quad (2)$$

$$\frac{a(b-c)}{\sin \frac{A}{2} \tan \frac{A}{2}} = \frac{(b+c)^2 \tan\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B-C}{2}\right)} \quad (5)$$

$$a(b-c) \operatorname{cosec} \frac{A}{2} \cot \frac{A}{2} = (b+c)^2 \tan\left(\frac{B-C}{2}\right) \sec\left(\frac{B-C}{2}\right) \quad (10)$$



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k} \quad (1)$$

In triangle ABD by
sin rule

$$\frac{m}{\sin B} = \frac{\frac{a}{2}}{\sin \theta} \quad (10)$$

$$2m \sin \theta = a \sin B \quad (1)$$

In triangle ACD

$$\frac{m}{\sin C} = \frac{\frac{a}{2}}{\sin \phi} \quad (10)$$

$$2m \sin \phi = a \sin C \quad (2)$$

(1) - (2) ⇒

$$2m(\sin \theta - \sin \phi) = a(\sin B - \sin C) \quad (5)$$

$$2m(\sin \theta - \sin \phi) = k \sin A(\sin B - \sin C) \quad (5) \quad (\text{by } *)$$

$$2m \cdot 2 \cos\left(\frac{\theta+\phi}{2}\right) \sin\left(\frac{\theta-\phi}{2}\right) = 2 \sin \frac{A}{2} \cos \frac{A}{2} (b-c) \quad (10)$$

$$4m \cos \frac{A}{2} \sin\left(\frac{\theta-\phi}{2}\right) = 2 \sin \frac{A}{2} \cos \frac{A}{2} (b-c) \quad (5) \quad (\text{by } *)$$

$$2m \sin\left(\frac{\theta-\phi}{2}\right) = (b-c) \sin \frac{A}{2} \quad (5) \quad (55)$$

$$c) \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4} \quad (1)$$

$$\alpha = \tan^{-1}(2x) \quad (5)$$

$$\beta = \tan^{-1}(3x) \quad (5)$$

$$(1) \Rightarrow \alpha + \beta = \frac{\pi}{4}$$

$$\tan(\alpha + \beta) = 1 \quad (5)$$

$$\frac{2x+3x}{1-(2x)(3x)} = 1 \quad (5)$$

$$1 - (2x)(3x)$$

$$6x^2 + 5x - 1 = 0 \quad (5)$$

$$(6x-1)(x+1) = 0 \quad (5)$$

$$x = \frac{1}{6} \text{ or } x = -1$$

$$\text{soln } x = \frac{1}{6} \quad (\because x > 0) \quad (30)$$

14.

$$a) \cos^2 \theta + \cos^2 \left(\theta + \frac{2\pi}{3} \right) + \cos^2 \left(\theta - \frac{2\pi}{3} \right)$$

$$= \frac{1 - \cos 2\theta}{2} + \frac{1 - \cos(2\theta + \frac{4\pi}{3})}{2} + \frac{1 - \cos(2\theta - \frac{4\pi}{3})}{2} \quad (15)$$

$$= \frac{3}{2} - \frac{1}{2} \left\{ \cos 2\theta + \cos(2\theta + \frac{4\pi}{3}) + \cos(2\theta - \frac{4\pi}{3}) \right\} \quad (5)$$

$$= \frac{3}{2} - \frac{1}{2} \left\{ \cos 2\theta + 2 \cos 2\theta \cos \frac{4\pi}{3} \right\} \quad (10)$$

$$= \frac{3}{2} - \frac{1}{2} \left\{ \cos 2\theta + 2 \cos 2\theta \left(-\frac{1}{2} \right) \right\} \quad (5)$$

$$= \frac{3}{2}$$

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$$b) 3 \sin 2\theta + 2 \sin^2 \theta = 2$$

$$3 \sin 2\theta + 1 - \cos 2\theta = 2 \quad (5)$$

$$3 \sin 2\theta - \cos 2\theta = 1$$

$$\frac{3}{\sqrt{10}} \sin 2\theta - \frac{1}{\sqrt{10}} \cos 2\theta = \frac{1}{\sqrt{10}} \quad (5)$$

$$\cos \alpha \sin 2\theta - \sin \alpha \cos 2\theta = \sin \alpha \quad (5) \quad \text{Here } \sin \alpha = \frac{1}{\sqrt{10}}$$

$$\sin(2\theta - \alpha) = \sin \alpha \quad (5) \quad \cos \alpha = \frac{3}{\sqrt{10}}$$

$$2\theta - \alpha = n\pi + (-1)^n \alpha, \quad n \in \mathbb{Z} \quad (5)$$

$$\text{If } n \text{ even } n = n_1 \text{ (say)}$$

$$2\theta - \alpha = n_1\pi + \alpha$$

$$\theta = \frac{n_1\pi}{2} + \alpha \quad (5)$$

$$\text{If } n \text{ odd } n = n_2 \text{ (say)}$$

$$2\theta - \alpha = n_2\pi - \alpha$$

$$\theta = \frac{n_2\pi}{2} \quad (5)$$

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$$c) a) \tan^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 0$$

$$(\tan \theta - 1)(\tan \theta - \sqrt{3}) = 0 \quad (5)$$

$$\tan \theta - 1 = 0 \text{ or } \tan \theta - \sqrt{3} = 0$$

$$\tan \theta = \tan \frac{\pi}{4} \quad (5) \quad \tan \theta = \tan \frac{\pi}{3} \quad (5)$$

$$\theta = n_1\pi + \frac{\pi}{4} \quad (5) \quad n_1 \in \mathbb{Z}$$

$$\theta = n_2\pi + \frac{\pi}{3} \quad (5) \quad n_2 \in \mathbb{Z}$$

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$$(ii) \cos 6x \cos 2x - \cos 3x \cos x = 0$$

$$2 \cos 6x \cos 2x - 2 \cos 3x \cos x = 0$$

$$(\cos 8x + \cos 4x) - (\cos 4x + \cos 2x) = 0 \quad (10)$$

$$\cos 8x = \cos 2x \quad (5)$$

$$8x = 2n_1\pi \pm 2x, \quad n_1 \in \mathbb{Z}$$

$$(+) 8x = 2n_1\pi + 2x$$

$$x = \frac{n_1\pi}{3}, \quad n_1 \in \mathbb{Z} \quad (5)$$

$$(-) 8x = 2n_2\pi - 2x$$

$$x = \frac{n_2\pi}{5}, \quad n_2 \in \mathbb{Z} \quad (5)$$

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$$d) 9 \cos(2 \sin^{-1} x) = 1 \quad (1)$$

$$\theta = \sin^{-1} x \text{ (say)}$$

$$\sin \theta = x \quad (5)$$

$$\Rightarrow 9 \cos 2\theta = 1 \quad (5)$$

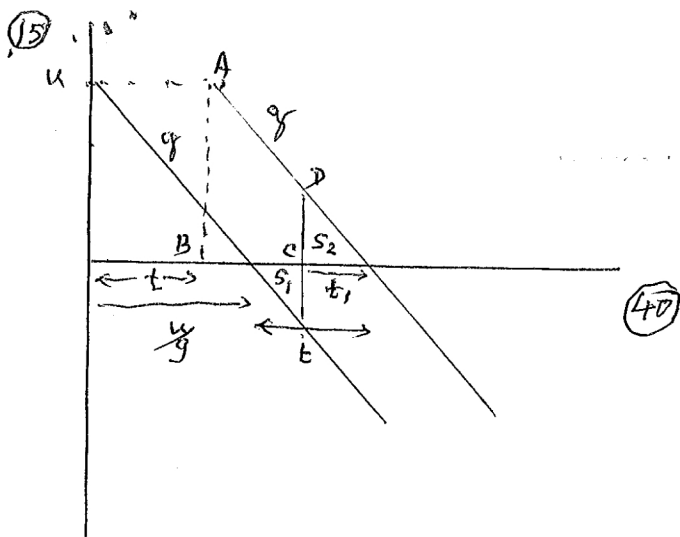
$$9[1 - 2 \sin^2 \theta] = 1 \quad (5)$$

$$9[1 - 2x^2] = 1 \quad (5)$$

$$x^2 = \frac{4}{9} \quad (5)$$

$$x = \frac{2}{3} \quad (\because x > 0) \quad (5)$$

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$$\text{Area } ABCD + S_2 = \frac{u^2}{2g} \quad (10)$$

$$\text{Area } ABCD = \frac{u^2}{2g} - S_1 \quad (10)$$

$$\Rightarrow S_2 = S_1 \quad (10)$$

$$\therefore t_1 = \frac{t}{2} \quad (10)$$

$$i \quad \frac{u}{g} + \frac{t}{2} \quad (10)$$

$$ii \quad \text{Area } ABCD = \frac{u^2}{2g} - \frac{1}{2}g\left(t - \frac{t}{2}\right)^2 \quad (10)$$

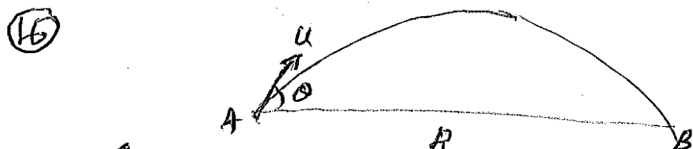
$$H = \frac{u^2}{2g} - \frac{1}{2}g\frac{t^2}{4} \quad (10)$$

$$c \quad \frac{u}{g} + \frac{t}{2} = \frac{4u}{3g} \quad (10)$$

$$t = \frac{2u}{3g} \quad (10)$$

$$ii \quad H = \frac{u^2}{2g} - \frac{1}{8}g\left(\frac{2u}{3g}\right)^2 \quad (10)$$

$$= \frac{4}{9}\frac{u^2}{g} \quad (10)$$



$$i \quad 0 = u \sin \theta \cdot t - \frac{1}{2}gt^2 \quad (10)$$

$$t = \frac{2u \sin \theta}{g} \quad (10)$$

$$\rightarrow R = u \cos \theta \cdot \frac{2u \sin \theta}{g} \quad (10)$$

$$= \frac{2u^2 \sin 2\theta}{g} \quad (10)$$

$$= \frac{u^2}{g} \sin 2\left(\frac{\pi}{2} - \theta\right) \quad (10)$$

directions are $\theta, \frac{\pi}{2} - \theta$ (10)

for θ

$$v = u + at$$

$$0 = u \sin \theta - gt$$

$$t_1 = \frac{u \sin \theta}{g} \quad (10)$$

$$v^2 = u^2 + 2as$$

$$0 = (u \sin \theta)^2 - 2gh_1 \quad (10)$$

$$h_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{for } \frac{\pi}{2} - \theta, \quad t_2 = \frac{u \cos \theta}{g} \quad (10)$$

$$h_2 = \frac{u^2 \cos^2 \theta}{2g} \quad (10)$$

$$R = 2u^2 \sin \theta \cos \theta$$

$$= 2 \frac{u^2}{g} \cdot \frac{\sqrt{2gh_1}}{u} \cdot \frac{\sqrt{2gh_2}}{u} \quad (10)$$

$$= 4 \sqrt{h_1 h_2} \quad (10)$$

$$R = 2 \frac{u^2}{g} \cdot \frac{gt_1}{u} \cdot \frac{gt_2}{u} \quad (10)$$

$$= 2gt_1 t_2$$

$$R = 4 \sqrt{h_1 h_2} = 2gt_1 t_2 \quad (10)$$

(17) (a) Theory

$$\vec{OD} = \frac{1}{2}(\vec{a} + \vec{b}) \quad (5)$$

$$\vec{AC} = \frac{1}{2}(\vec{b} - \vec{a}) \quad (5)$$

$$\vec{MC} = \lambda(\frac{1}{2}(\vec{b} - \vec{a}))$$

$$\vec{OM} = \frac{\mu}{2}(\vec{a} + \vec{b})$$

$$\vec{OM} + \vec{MC} = \vec{OC}$$

$$\frac{\mu}{2}(\vec{a} + \vec{b}) + \lambda(\frac{1}{2}(\vec{b} - \vec{a})) = \frac{1}{2}\vec{b} \quad (10)$$

$$\left(\frac{\mu}{2} - \lambda\right)\vec{a} + \left(\frac{\mu}{2} + \frac{\lambda}{2} - \frac{1}{2}\right)\vec{b} = 0$$

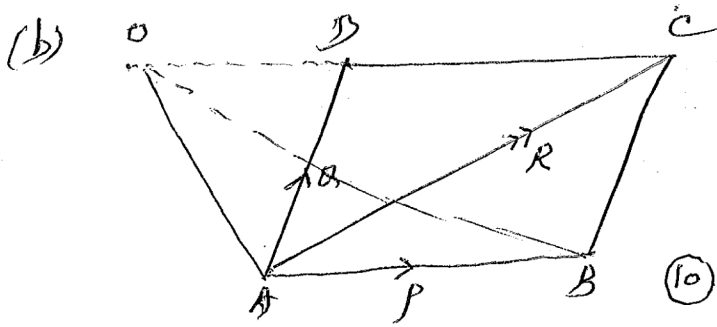
$$\frac{\mu}{2} - \lambda = 0 \quad \& \quad \frac{\mu}{2} + \frac{\lambda}{2} - \frac{1}{2} = 0$$

$$\lambda = \frac{1}{3}, \mu = \frac{2}{3}$$

M is dividing the medians 2:1 (10)

(b)

[07]



or
moment of P + moment of Q

$$= 2\Delta OAB + 2\Delta OAD$$

$$= 2\Delta ABC + 2\Delta OAD$$

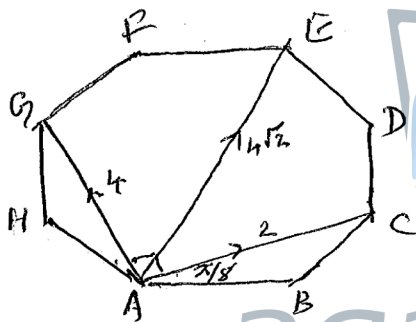
$$= 2\Delta ADC + 2\Delta OAD$$

$$= 2(\Delta ADC + \Delta OAD)$$

$$= 2\Delta OAC$$

$$= \text{moment of } R.$$

(c)



$$\vec{AC} \cdot x = 2 + 4\sqrt{2} \cos 45^\circ$$

$$= 6$$

$$\vec{AG} \cdot y = 4 + 4\sqrt{2} \sin 45^\circ$$

$$= 8$$

$$R^2 = x^2 + y^2$$

$$= 6^2 + 8^2$$

$$R = 10$$

$$\tan \alpha = \frac{8}{6} = \frac{4}{3}$$

Resultant makes $(\frac{\pi}{8} + \tan^{-1} \frac{4}{3})$ with AB.