# G.C.E. A/L Examination March - 2018

## Conducted by Field Work Centre, Thondaimanaru
## In Collaboration with
FWC
## Provincial Department of Education Northern Province.

| Grade :- 12 (2019) | Combined Mathematics | Marking Scheme |
|---|---|---|

1. $f(x) = 3x^2 + 6kx + 4k^2 + 1$

$= 3(x^2 + 2kx + k^2) + k^2 + 1$ (5)

$= 3(x+k)^2 + k^2 + 1$ (5)

$(f(x))_{min} = k^2 + 1$ (5)

$k^2 + 1 = 5$ (5)

$k^2 = 4$

$k = \pm 2$ (5) $\boxed{25}$

2. $\dfrac{x(x^2+x-4)}{x^2+1} \geq -x$

$x(x^2+x-4) \geq -x(x^2+1)$ (5) $\because x^2+1>0$

$E = x(x-1)(2x+3) \geq 0$ (5)

| | $x<-\frac{3}{2}$ | $-\frac{3}{2}<x<0$ | $0<x<1$ | $x>1$ |
|---|---|---|---|---|
| $x$ | $(-)$ | $(-)$ | $(+)$ | $(+)$ |
| $x-1$ | $(-)$ | $(-)$ | $(-)$ | $(+)$ |
| $2x+3$ | $(-)$ | $(+)$ | $(+)$ | $(+)$ |
| E | $(-)$ | $(+)$ | $(-)$ | $(+)$ |

(10)

$-\frac{3}{2} \leq x \leq 0$ or $x \geq 1$ (5) $\boxed{25}$

3. $\log_a a^2b^2 + \log_b a^2b^2 = 9$

$\Rightarrow 2\log_a ab + 2\log_b ab = 9$ (5)

$\Rightarrow 2(\log_a a + \log_a b) + 2(\log_b a + \log_b b) = 9$ (5)

$\Rightarrow 2\log_a b + 2\log_b a = 5$

$\Rightarrow 2\log_a b + \dfrac{2}{\log_a b} = 5$ (5)

$\Rightarrow 2t + \dfrac{2}{t} = 5$ ; (5)

$\Rightarrow 2t^2 - 5t + 2 = 0$

$\Rightarrow (2t-1)(t-2) = 0$

$\Rightarrow t = \frac{1}{2}$ or $t = 2$

$\log_a b = \frac{1}{2}$ or $\log_a b = 2$ (5) $\boxed{25}$

4. $\lim\limits_{x\to 0} \dfrac{1 + \cos x - \cos 2x - \cos 3x}{x \sin x}$

$= \lim \dfrac{2\sin^2 x + 2\sin x \sin 2x}{x \sin x}$ (10)

$= 2\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right) + \lim\limits_{x\to 0}\left(\dfrac{\sin 2x}{2x}\right)\cdot 2\cdot 2$ (10)

$= 2 + 4$ (5) $= 6$

5. $\dfrac{1 + \sin 2\theta - \cos 2\theta}{1 + \sin 2\theta + \cos 2\theta}$

$= \dfrac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta}$ (5)

$= \dfrac{\sin\theta(\sin\theta + \cos\theta)}{\cos\theta(\cos\theta + \sin\theta)}$ (5)

$= \tan\theta$ (5)

$\theta = \frac{\pi}{8} \Rightarrow \dfrac{1 + \sin\frac{\pi}{4} - \cos\frac{\pi}{4}}{1 + \sin\frac{\pi}{4} + \cos\frac{\pi}{4}} = \tan\frac{\pi}{8}$ (5)

$\Rightarrow \dfrac{1}{1+\sqrt{2}} = \tan\frac{\pi}{8}$ (5)

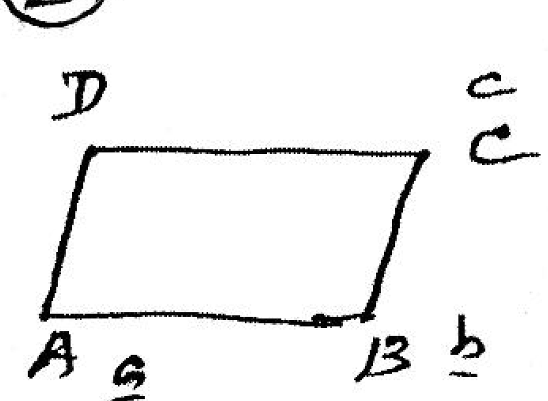
$\dfrac{\sqrt{2}-1}{1} = \tan\frac{\pi}{8}$.

6. $s = ut + \frac{1}{2}at^2 \downarrow$

$H_1 = \frac{1}{2}g t_1^2$ (10)

$H_2 = \frac{1}{2}g t_2^2$ (10)

$\dfrac{H_1}{H_2} = \dfrac{t_1^2}{t_2^2}$ (5)

7. 


$\overrightarrow{AD} = \overrightarrow{BC}$ (10)

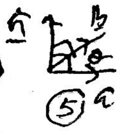
$\overrightarrow{AO} + \overrightarrow{OD} = \overrightarrow{BO} + \overrightarrow{OC}$ (10)

$-\underline{a} + \underline{d} = -\underline{b} + \underline{c}$

$\underline{d} = \underline{a} - \underline{b} + \underline{c}$ (5)

(8)

$\underline{a} \wedge \underline{b} = |\underline{a}||\underline{b}| \sin\theta \, \hat{\underline{n}}$ (10)



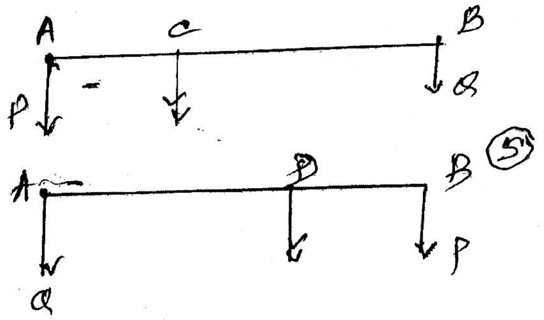
(5)

$|a \times b| = |a||b| \sin\theta \cdot 1$ (5)

$\hat{\underline{n}} = \dfrac{\underline{a} \times \underline{b}}{|a||b| \sin\theta}$ (5)

$= \dfrac{a \times b}{|a \times b|}$.

(9)



(5)

$\dfrac{AC}{CB} = \dfrac{Q}{P}$ (5)

$\dfrac{AC}{AC+CB} = \dfrac{Q}{P+Q}$ (5)

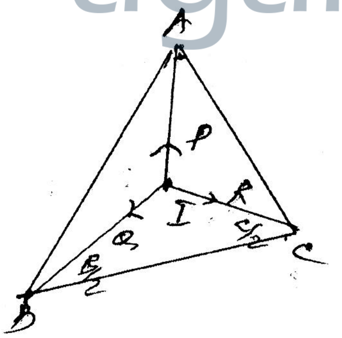
$AC = \dfrac{Q}{P+Q} \cdot AB$ (5)

|||ly $\dfrac{AD}{} = \dfrac{P}{P+Q} \cdot AB$ (5)

$CD = AD - AC$

$= \dfrac{(P-Q)}{P+Q} AB$

(10)



By Lamis Theorem (15)

$\dfrac{P}{\sin(\frac{\pi}{2}-\frac{A}{2})} = \dfrac{Q}{\sin(\frac{\pi}{2}-\frac{B}{2})} = \dfrac{R}{\sin(\frac{\pi}{2}-\frac{C}{2})}$

$\dfrac{P}{\cos\frac{A}{2}} = \dfrac{Q}{\cos\frac{B}{2}} = \dfrac{R}{\cos\frac{C}{2}}$ (10)

$P \sec\frac{A}{2} = Q \sec\frac{B}{2} = R \sec\frac{C}{2}$

(11) $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$

(5) $3x^2 - 2(a+b+c)x + ab+bc+ca = 0$

(10) $\Delta = 4(a+b+c)^2 - 4\cdot 3(ab+bc+ca)$

$= 4\left[a^2+b^2+c^2 - ab - bc - ca\right]$

(10) $= 2\left[(a-b)^2+(b-c)^2+(c-a)^2\right] \geq 0$.

$\Rightarrow \alpha, \beta$ real. (5)

Let $y = \frac{1}{x}$. $x = \alpha \Rightarrow y = \frac{1}{\alpha}$ (10)

$x = \beta \Rightarrow y = \frac{1}{\beta}$.

(5)

$\therefore 3\left(\frac{1}{y}\right)^2 - 2(a+b+c)\frac{1}{y} + ab+bc+ca = 0$

$(ab+bc+ca)y^2 - 2(a+b+c)y + 3 = 0$ (5)

b) $ax^2+bx+c = 0$ $\alpha, \beta$

$\alpha+\beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$ (10)

$b^2 - 4ac > 0$ & $\alpha\beta < 0$ (10)

$\alpha(x-\beta)^2 + \beta(x-\alpha)^2 = 0$

$\alpha(x^2+\beta^2-2\beta x) + \beta(x^2+\alpha^2-2\alpha x) = 0$ (5)

$(\alpha+\beta)x^2 - 4\alpha\beta x + \alpha\beta(\alpha+\beta) = 0$ (5)

$\Delta_1 = (4\alpha\beta)^2 - 4(\alpha+\beta)^2\alpha\beta > 0$ (10)

$(\because \alpha\beta < 0)$

Roots are real

Let $\alpha_1, \beta_1$ are roots (10)

$\alpha_1\beta_1 = \dfrac{\alpha\beta(\alpha+\beta)}{\alpha+\beta} = \alpha\beta < 0$

(c) $x^3 - 2x^2 + 2x - 2 = (x-2)(x+1)\phi(x) + Ax + B$ (10)

$x=2 \Rightarrow 2 = 2A+B$ (10)

$x=-1 \Rightarrow -7 = -A+B$

$A = 3, B = -4$ (10)

$x^3-2x^2+2x-2 = (x-2)(x+1)\phi(x) + 3x - 4$

$x^3-2x^2-x+2 = (x-2)(x+1)\phi(x)$ (10)

$= (x-2)(x+1)(px+q)$

$p = 1$ & $q = -1$ (10)

$\therefore x^3-2x^2-x+2 = (x-2)(x+1)(x-1)$

12. $(\sqrt{x}-\sqrt{y})^2 \geq 0$ (10)

$x+y-2\sqrt{xy} \geq 0$ (5)

$x+y \geq 2\sqrt{xy}$

(i) $a+b \geq 2\sqrt{ab}$

$b+c \geq 2\sqrt{bc}$ (15)

$c+a \geq 2\sqrt{ca}$

(5)

$(a+b)(b+c)(c+a) \geq 8\sqrt{a^2b^2c^2} = 8abc$

(ii) $\frac{a^2}{b}+b \geq 2a$

$\frac{b^2}{c}+c \geq 2b$ (10)

$\frac{c^2}{a}+a \geq 2c$

$\Rightarrow \frac{a^2}{b}+b+\frac{b^2}{c}+c+\frac{c^2}{a}+a \geq 2(a+b+c)$ (5)

$\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a} \geq a+b+c$

(b) $\log_a b = p$, $\log_b c = q$ (10)

$b = a^p$, $c = b^q$

$b^q = a^{pq} = c$ (10)

$a^{pq} = c$ (5)

$\log_a c = pq$ (10)

$= \log_a b \cdot \log_b c$

$\log_a b \cdot \log_b a = \log_a a = 1$ (10)

$\therefore \log_a b = \frac{1}{\log_b a}$

$p = \frac{\log_m x}{\log_a x}$

$= \log_m x \cdot \log_x a$ (10)

$= \log_m a$

Illy $q = \log_m b$ (5)

$r = \log_m \frac{1}{ab}$ (5)

$p+q+r = \log_m a + \log_m b + \log_m \frac{1}{ab}$

$= \log_m \left(ab \cdot \frac{1}{ab}\right)$ (10)

$= 0$

$p+q+r = 0$

$\Rightarrow (p+q)^3 = (-r)^3$ (5)

$p^3+3p^2q+3pq^2+q^3+r^3 = 0$

$p^3+q^3+r^3+3pq(p+q) = 0$ (10)

$p^3+q^3+r^3+3pq(-r) = 0$ (10)

$p^3+q^3+r^3 = 3pqr$.

13. (a)

I $y = \ln|x+\sqrt{1+x^2}|$ (15)

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{x}{\sqrt{1+x^2}}\right)$

(15)

$= \frac{1}{\sqrt{1+x^2}}$

II $y = a\cos^3\theta$, $x = a\sin^3\theta$

$\frac{dy}{d\theta} = a.3\cos^2\theta(-\sin\theta)$ (10)

$\frac{dx}{d\theta} = a.3\sin^2\theta(\cos\theta)$ (10)

$\frac{dy}{dx} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$ (10)

$= -\cot\theta$

III $y = x^{x^2+1}$

$\ln y = (x^2+1)\ln|x|$ (5)

$\frac{1}{y}\frac{dy}{dx} = \frac{x^2+1}{x} + \ln x \cdot 2x$ (10)

$\frac{dy}{dx} = \frac{1}{x}\left[x^2+1+2x^2\ln x\right]x^{x^2+1}$ (5)

$= x^{x^2}\left[x^2+1+2x^2\ln x\right]$ (10)

b) $y = \cos(\cos x)$

$\frac{dy}{dx} = -\sin(\cos x).(-\sin x)$ (10)

$= \sin x.\sin(\cos x)$

$\frac{d^2y}{dx^2} = \sin x\cos(\cos x)(-\sin x) + \sin(\cos x).\cos x$

$= -y\sin^2 x + \cos x.\sin(\cos x)$ (10)

$= -y\sin^2 x + \frac{\cos x}{\sin x}\frac{dy}{dx}$ (10)

$\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} + y\sin^2 x = 0$

$y = (\cos^{-1}x)^2$

$\frac{dy}{dx} = 2\cos^{-1}x \cdot \frac{1}{-\sqrt{1-x^2}}$ (10)

$\left(\frac{dy}{dx}\right)^2 = \frac{4}{1-x^2}\cdot(\cos^{-1}x)^2$ (5)

$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$ (5)

$(1-x^2)\,2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = 4\frac{dy}{dx}$ (10)

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

(14) $2(\sin 2x - \sin x) + \sqrt{3}(2\cos x - 1) = 0$

(10) $2\sin x(2\cos x - 1) + \sqrt{3}(2\cos x - 1) = 0$

(5) $(2\cos x - 1)(2\sin x + \sqrt{3}) = 0$

(10) $\cos x = \frac{1}{2}$ or $\sin x = -\frac{\sqrt{3}}{2}$

(5) $x = 2n\pi \pm \frac{\pi}{3}$ or $x = m\pi + (-1)^n\left(-\frac{\pi}{3}\right)$

$n, m \in Z$.

ii $\sin x + \cos x = \sin\frac{\pi}{12} + \cos\frac{\pi}{12}$

(10) $\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}\sin\frac{\pi}{12} + \frac{1}{\sqrt{2}}\cos\frac{\pi}{12}$

(10) $\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{6}$

$x - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{6}$

(10) $x = 2k\pi \pm \frac{\pi}{6} + \frac{\pi}{4}, \quad k \in Z.$

b) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k}$ (10)

$\frac{a}{b-c} = \frac{k\sin A}{k\sin B - k\sin C}$ (10)

$= \frac{\sin A}{\sin B - \sin C}$

$= \frac{2\sin A/2\cos A/2}{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}$ (10)

$= \frac{\cos A/2}{\sin\frac{B-C}{2}}$ (5)

ii $\frac{b+c}{b-c} = \frac{\sin B + \sin C}{\sin B - \sin C}$ (10)

$= \frac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}$ (10)

$= \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}\cdot\cot\frac{B-C}{2}$ (5)

$\frac{b+c}{b-c}\cdot\tan\frac{A}{2} = \cot\frac{B-C}{2}$

c) $\underbrace{\tan^{-1}2x}_{\alpha} + \underbrace{\tan^{-1}3x}_{\beta} = \frac{\pi}{4}$

$\tan\alpha = 2x, \quad \tan\beta = 3x$ (10)

$\tan(\alpha+\beta) = \tan\frac{\pi}{4}$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$

$\frac{2x + 3x}{1 - 2x\cdot 3x} = 1$ (10)

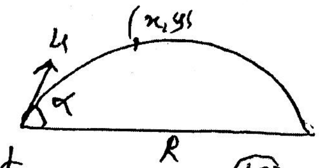
$5x = 1 - 6x^2$

$6x^2 + 5x - 1 = 0$

$(6x - 1)(x + 1) = 0$ (5)

$x = \frac{1}{6}$ (5) $(\because x > 0)$

(15)




$\rightarrow x = u\cos\alpha\cdot t$ (10)

$\uparrow y = u\sin\alpha\cdot t - \frac{1}{2}gt^2$ (10) ~~(5)~~

$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ (10)

$= x\tan\alpha\left[1 - \frac{gx}{2u^2\sin\alpha\cos\alpha}\right]$ (10)

$y = 0, \quad x = R$

$\therefore R = \frac{2u^2\sin\alpha\cos\alpha}{g}$ (10)

$\therefore y = x\tan\alpha\left(1 - \frac{x}{R}\right)$ (10)

$= -\frac{x^2\tan\alpha}{R} + x\tan\alpha$ (10)

$= -\frac{\tan\alpha}{R}\left(x - \frac{R}{2}\right)^2 + \frac{R}{4}\tan\alpha$ (10)

$$R'' = 2\frac{(ku)^2 \sin(\alpha-\lambda)\cos(\alpha-\lambda)}{g} \quad (10)$$

$$\frac{R'}{R} = \frac{k^2 \sin(\alpha-\lambda)\cos(\alpha-\lambda)}{\sin\alpha\cos\alpha} \quad (10)$$

Maximum Heights are same

$$\frac{R}{4}\tan\alpha = \frac{R'}{4}\tan(\alpha-\lambda) \quad (10)$$

$$R\tan\alpha = R'\tan(\alpha-\lambda)$$

$$\frac{\tan\alpha}{\tan(\alpha-\lambda)} = \frac{k^2\sin(\alpha-\lambda)\cos(\alpha-\lambda)}{\sin\alpha\cos\alpha} \quad (10)$$

$$\sin^2\alpha = k^2\sin^2(\alpha-\lambda)$$

$$\sin\alpha = k\sin(\alpha-\lambda) \quad (10)$$

$$\sin\alpha = k(\sin\alpha\cos\lambda - \cos\alpha\sin\lambda) \quad (10)$$

$$\csc\lambda = k(\cot\lambda - \cot\alpha) \quad (10)$$

16 a) $\lambda \underline{a} + \mu \underline{b} = 0$

let $\lambda \neq 0 \Rightarrow \underline{a} + \frac{\mu}{\lambda}\underline{b} = 0$ (10)

(10) $\underline{a} = -\frac{\mu}{\lambda}\underline{b}$

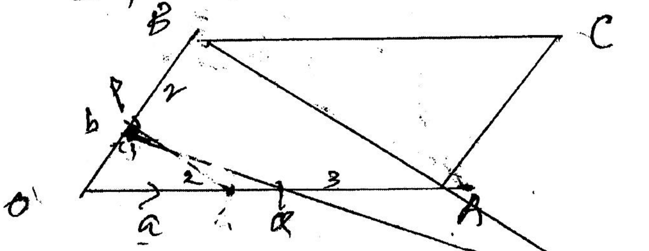
$\Rightarrow \underline{a} \parallel \underline{b}$ (10)

but $\underline{a} \nparallel \underline{b}$ (5)

$\therefore \lambda = 0$ (5)

$\Rightarrow \mu\underline{b} = 0$ (5)

$\underline{b} \neq 0 \Rightarrow \mu = 0$




$\vec{BA} = \underline{a} - \underline{b}$ (5)

$\vec{PQ} = \vec{PO} + \vec{OQ}$

$= \frac{1}{3}\vec{BO} + \frac{2}{5}\vec{OA}$

$= \frac{2}{5}\underline{a} - \frac{1}{3}\underline{b}$ (10)

$\vec{PR} = \lambda\vec{PQ}$

$= \lambda\left(\frac{2}{5}\underline{a} - \frac{1}{3}\underline{b}\right)$ (5)

$\vec{BR} = \mu\vec{BA}$

$= \mu(\underline{a} - \underline{b})$ (5)

$\vec{BP} = \vec{BR} + \vec{RP}$

$\frac{2}{3}\underline{b} = \mu(\underline{a} - \underline{b}) + \lambda\left(\frac{1}{3}\underline{b} - \frac{2}{5}\underline{a}\right)$ (10)

$\left(\mu - \frac{2\lambda}{5}\right)\underline{a} + \left(\frac{\lambda}{3} - \mu - \frac{2}{3}\right)\underline{b} = 0$

$\Rightarrow \mu - \frac{2\lambda}{5} = 0$ and $\frac{\lambda}{3} - \mu - \frac{2}{3} = 0$ (10)

$\mu = \frac{2\lambda}{5}$ and $\frac{\lambda}{3} - \frac{2\lambda}{5} - \frac{2}{3} = 0$

$-\lambda - 10 = 0$

$\lambda = 10$

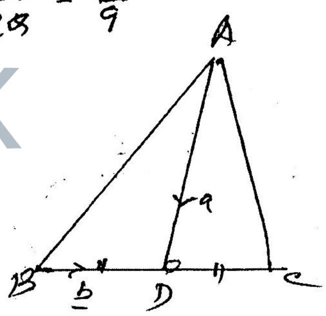
$\mu = 4.$ (10)

$\vec{BR} = 4\vec{BA}$

$\frac{BR}{BA} = \frac{4}{1} \Rightarrow \frac{BR}{RA} = \frac{4}{3}$ (10)

$\vec{PR} = 10\vec{PQ}$ (10)

$\frac{PR}{PQ} = \frac{10}{1} \Rightarrow \frac{PR}{RQ} = \frac{10}{9}$




$\vec{AB} = \underline{a} - \underline{b}$ (10)

$\vec{AC} = \underline{a} + \underline{b}$ (10)

$AB^2 = |\underline{a} - \underline{b}|^2 = (\underline{a} - \underline{b})\cdot(\underline{a} - \underline{b})$ (10)

$= a^2 + b^2 - 2\underline{a}\cdot\underline{b}$

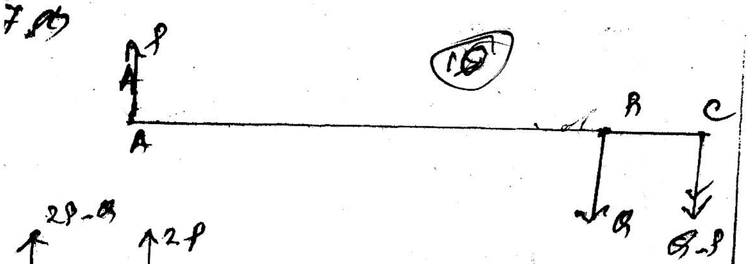
$AC^2 = |\underline{a} + \underline{b}|^2 = (\underline{a} + \underline{b})\cdot(\underline{a} + \underline{b})$ (10)

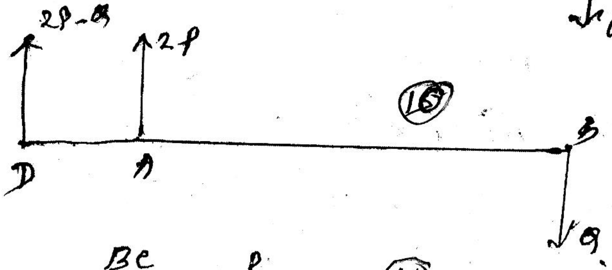
$= a^2 + b^2 + 2\underline{a}\cdot\underline{b}$

$AB^2 + AC^2 = 2a^2 + 2b^2$ (10)

$= 2AD^2 + 2BD^2.$

17 (b)




$$\frac{BC}{CA} = \frac{P}{Q}$$ 10

$$\Rightarrow \frac{BC}{AB} = \frac{P}{Q-P} \longrightarrow ①$$ 10

$$\frac{BD}{AD} = \frac{2P}{Q}$$ 10

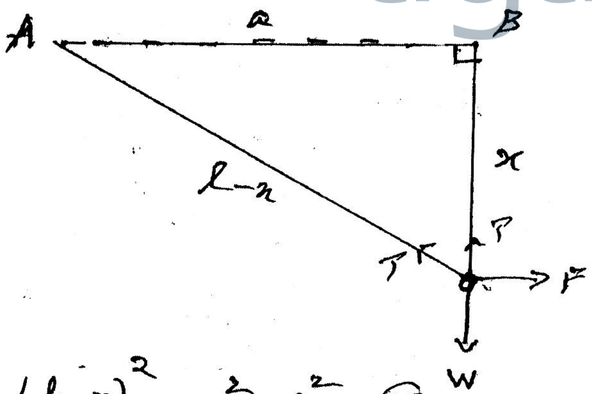
$$\Rightarrow \frac{BD}{AB} = \frac{2P}{2P-Q} \longrightarrow ②$$ 10

$$①+② \Rightarrow \frac{2P}{2P-Q} = \frac{P}{Q-P}$$ 10

$$2P - Q = 2(Q-P)$$

$$4P = 3Q$$ 10

$$\frac{P}{Q} = \frac{3}{4}$$



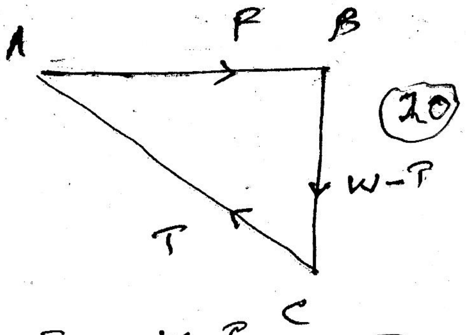

$$(l-x)^2 = x^2 + a^2$$ 10

$$l^2 - 2lx = a^2$$ 10

$$x = \frac{l^2 - a^2}{2l}$$

$$l - x = l - \frac{l^2 - a^2}{2l}$$ 10

$$= \frac{l^2 + a^2}{2l}$$




$$\frac{F}{AB} = \frac{T}{AC} = \frac{W-T}{BC}$$ 10

$$\frac{F}{a} = \frac{T}{AC} = \frac{W-T}{BC}$$

$$\frac{T+W-T}{AC+BC} = \frac{T}{AC}$$

$$\frac{W}{l} = \frac{T}{AC}$$

$$T = \frac{W}{l}\left(\frac{l^2+a^2}{2l}\right)$$ 5

$$F = T \cdot \frac{a}{AC}$$

$$= \frac{W}{l} \cdot \left(\frac{l^2+a^2}{2l}\right) \cdot \frac{a}{\frac{l^2+a^2}{2l}}$$

$$= \frac{Wa}{2l}$$ 5