$f(x) = 2x^2 - 12x + 22$

$= 2(x^2 - 6x) + 22$

$= 2(x-3)^2 + 4$ (10)

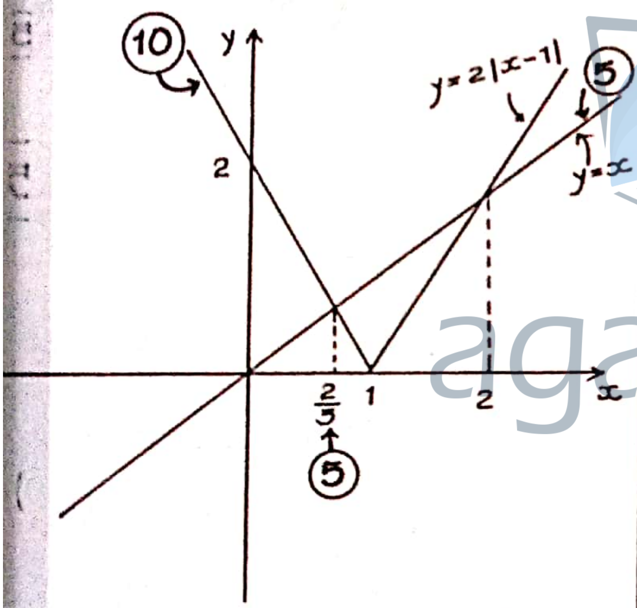
$= a(x-b)^2 + c$, where

$a = 2$
$b = 3$
$c = 4$ (5)

$f(x)\Big|_{min} = 4$ (5)

$\dfrac{1}{f(x)}\Big|_{max} = \dfrac{1}{4}$ (5)

25



$2(x-1) = x$
$x = 2$

$\left.\begin{array}{l} y = -2x+2 \\ y = x \end{array}\right\} x = \dfrac{2}{3}$

The solution is

$\dfrac{2}{3} < x < 2$. (5)

25

$\lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x \sin 2x}$

$= \lim\limits_{x\to 0} \dfrac{(1-\cos x)(1+\cos x+\cos^2 x)}{x \; 2\sin x \cos x}$ (5)

$= \lim\limits_{x\to 0} \dfrac{(1-\cos^2 x)(1+\cos x+\cos^2 x)}{x \; 2\sin x \cos x \; (1+\cos x)}$ (5)

$= \lim\limits_{x\to 0} \dfrac{\sin x \; (1+\cos x+\cos^2 x)}{x \; 2\cos x \; (1+\cos x)}$ (5)

$= \lim\limits_{x\to 0} \dfrac{\sin x}{x} \cdot \lim\limits_{x\to 0} \dfrac{1+\cos x+\cos^2 x}{2\cos x(1+\cos x)}$ (5)

$= 1 \times \dfrac{3}{2(2)}$ (5)

$= \dfrac{3}{4}$

25

(04) $\dfrac{1}{2}\left\| \begin{matrix} k & 3k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{matrix} \right\|$ (10)

$= \dfrac{1}{2}\left| k(-2) - 3k(4) + 1(10) \right|$

$= \dfrac{1}{2}\left|14k - 10\right|$

$= |7k - 5|$ (5)

If A, B and C are collinear,

$\Delta ABC = 0$ (5)

$7k - 5 = 0$

$k = \dfrac{5}{7}$ (5)

25

(05) $A + C = 2B$

$180 - B = 2B$ (5)

$3B = 180^\circ$

$B = 60^\circ$ (5)

$$\frac{1}{2} = \frac{c^2+10^2-9^2}{2c(10)} \quad 5$$

$$c^2-10c+19=0 \quad 5$$

$$c = 5 \pm \sqrt{6} \quad 5$$

25

(06) $x_1 = \left(up + \frac{1}{2}ap^2\right) - \left(u(p-1) + \frac{1}{2}a(p-1)^2\right)$ 10

$= u + \frac{1}{2}a(2p-1)$

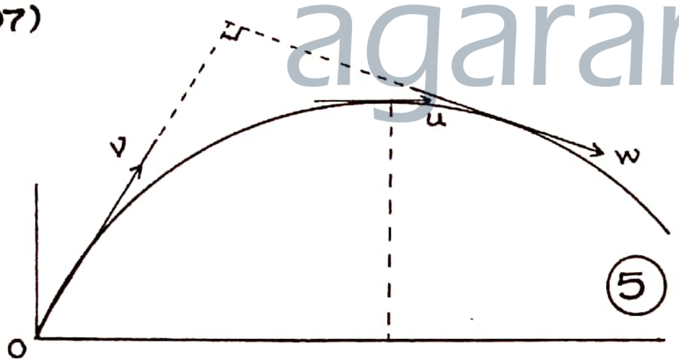
$x_2 = u + \frac{1}{2}a\left(2(p+q)-1\right)$ 10

$x_2 - x_1 = aq$ 5

$a = \dfrac{x_2 - x_1}{q}$

25

(07)



5

$v\cos\theta = u$ ..... (1) 5

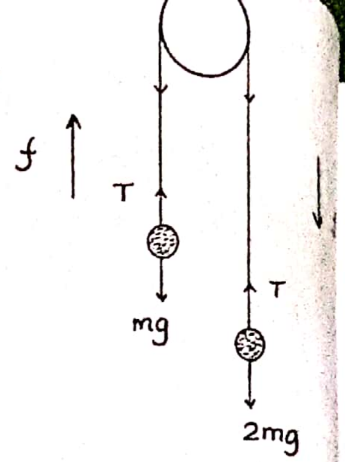
$w\sin\theta = u$ ..... (2) 5

$\dfrac{\cos\theta}{u} = \dfrac{1}{v}$ 5

$\dfrac{\sin\theta}{u} = \dfrac{1}{w}$ 5

$\Rightarrow \dfrac{1}{v^2} + \dfrac{1}{w^2} = \dfrac{1}{u^2}$

25



2m : $2mg - T = 2mf$

m : $T - mg = mf$

$mg = 3mf$

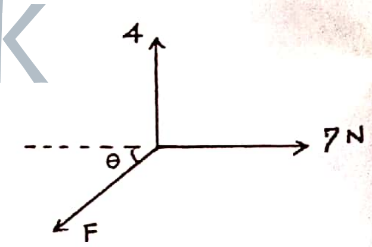
$f = \frac{g}{3}$ 5

$T = mg + mf$

$= mg + m\frac{g}{3}$

$= \frac{4}{3}mg$ 5

(09)



$\leftarrow$ $F\cos\theta = 7$ 5

$\downarrow$ $F\sin\theta = 4$ 5

$\tan\theta = \frac{4}{7}$ 5

$F^2 = 7^2 + 4^2$ 5

$F = \sqrt{65}$ N 5

$\overrightarrow{AC} = \underline{a}+\underline{b}$ (5)

$\overrightarrow{DB} = \underline{a}-\underline{b}$ (5)

$(\underline{a}+\underline{b})\cdot(\underline{a}-\underline{b}) = 0$ (5)

$|\underline{a}|^2 = |\underline{b}|^2$ (5)

$|\underline{a}| = |\underline{b}|$

$AB = BC$ (5)

∴ ABCD is a rhombus.

25

$f(x) = 5x^3 + ax^2 + x + b$

$f(1) = 0$ (5)

$5 + a + 1 + b = 0$

$a + b = -6 \;....(1)$ (5)

$f(2) = 3$ (5)

$40 + 4a + 2 + b = 3$

$4a + b = -39 \;....(2)$ (5)

$3a = -33$

$a = -11$ (5)

$b = 5$ (5)

$f(x) = (x-2)^2\phi(x) + Ax + B$ (5)

$f(2) = 2A + B$ (5)

$2A + B = 3$

$f'(x) = (x-2)^2\phi'(x) + \phi(x)2(x-2) + A$ (5)

when $x = 2$ $f'(2) = A$ (5)

$f'(x) = 15x^2 - 11(2x) + 1$ (5)

$f'(2) = 60 - 44 + 1$

$= 17$ (5)

$A = 17$ (5)

$B = -31$ (5)

∴ The remainder is $17x - 31$. (5)

75

(b) $g(x) = x^2 - (3a+1)x + (a+1)(a-2) = 0$

$\Delta = (3a+1)^2 - 4(1)(a+1)(a-2)$ (10)

$= 5a^2 + 10a + 9$ (5)

$= 5(a+1)^2 + 4 > 0$ (5)

∴ The roots are real and distinct.

$\alpha + \beta = 3a + 1$, $\alpha\beta = (a+1)(a-2)$ (10)

$|\alpha - \beta|^2 = (\alpha+\beta)^2 - 4\alpha\beta$ (5)

$= (3a+1)^2 - 4(a+1)(a-2)$ (5)

$= 5(a+1)^2 + 4$ (5)

$|\alpha - \beta| \geq 2$ (5)

∴ The difference between the roots is equal to 2 or greater than 2. (5)

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ (5)

$= \frac{(3a+1)^2 - 2(a+1)(a-2)}{(a+1)(a-2)}$

$= \frac{7a^2 - 4a + 5}{(a+1)(a-2)}$ (5)

$\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$ (5)

∴ The equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$x^2 - \left(\frac{7a^2 - 4a + 5}{(a+1)(a-2)}\right)x + 1 = 0.$

(12) (a) $y = (x-2)^2(x+3)$

$$\frac{dy}{dx} = (x-2)^2 + (x+3)\,2(x-2) \quad (5)$$

$$= (x-2)(3x+4) \quad (5)$$

When $\dfrac{dy}{dx} = 0$ $\quad x = 2$ or $x = -\dfrac{4}{3}$ (10)

When $x = 2$ $\quad y = 0$

When $x = -\dfrac{4}{3}$ $\quad y = \dfrac{500}{27}$

$-\frac{4}{3}$		2
$\frac{dy}{dx} > 0$ ↗	$\frac{dy}{dx} < 0$ ↘	$\frac{dy}{dx} > 0$ ↗

(15)

At $x = -\dfrac{4}{3}$ $\quad y$ maximum

$\left(-\dfrac{4}{3}, \dfrac{500}{27}\right)$ is maximum point. (5)

At $x = 2$ $\quad y$ minimum

$(2, 0)$ is minimum point. (5)

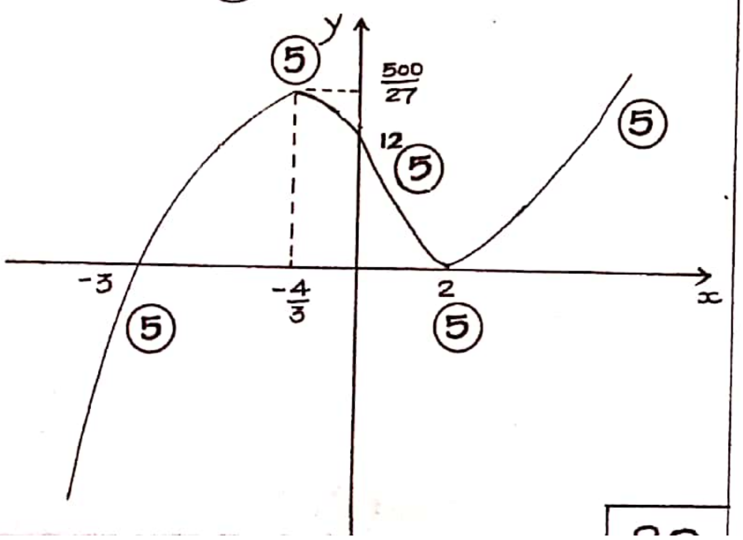
when $x = 0$ $\quad y = 12$

when $y = 0$ $\quad x = 2$ or $x = -3$ (5)

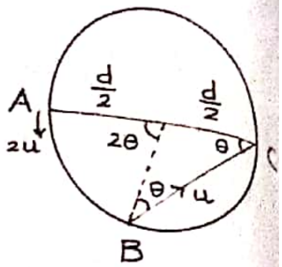
$$\lim_{x \to \pm\infty} (x-2)^2(x+3)$$

$$= \lim_{x \to \pm\infty} x^3\left(1 - \frac{2}{x}\right)\left(1 + \frac{3}{x}\right)$$

$$= \pm\infty \quad (5)$$



(b)



Arc AB $= d\theta$, BC $=$

The total time taken to C from A is

$$T = \frac{d\theta}{2u} + \frac{d\cos\theta}{u}$$

$$= \frac{d}{2u}\{\theta + 2\cos\theta\}$$

$$\frac{dT}{d\theta} = \frac{d}{2u}(1 - 2\sin\theta)$$

$$= -\frac{d}{u}\left(\sin\theta - \frac{1}{2}\right)$$

$$\frac{dT}{d\theta} = 0 \Rightarrow \sin\theta = \frac{1}{2} \quad (5)$$

$$\theta = \frac{\pi}{6}$$

$0 \le \theta < \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	
$\frac{dT}{d\theta} > 0$	$\frac{dT}{d\theta} = 0$	

$\therefore$ At $\theta = \dfrac{\pi}{6}$ $\quad T$ maxi

$$T_{max} = \frac{d}{2u}\left(\frac{\pi}{6} + 2\cos\right.$$

$$= \frac{d}{2u}\left(\frac{\pi}{6} + \sqrt{3}\right)$$

$$= \frac{d}{12u}(\pi + 6\sqrt{3})$$

when $\theta = 0$ $\quad T = \dfrac{d}{u}$ (5

when $\theta = \dfrac{\pi}{2}$ $\quad T = \dfrac{d}{2u} \cdot \dfrac{\pi}{2}$

$$= \frac{\pi}{4u}$$

The minimum value of T

$\dfrac{\pi d}{4u}$ sec. (5)

(13) (a) (i) $a^2-(b-c)^2\cos^2\frac{A}{2}$

$= a^2-(b-c)^2\left(1-\sin^2\frac{A}{2}\right)$ 5

$= a^2-b^2-c^2+2bc+(b^2+c^2-2bc)\sin^2\frac{A}{2}$ 5

$= -2bc\cos A+2bc+(b^2+c^2-2bc)\sin^2\frac{A}{2}$ 5

$= 2bc\cdot 2\sin^2\frac{A}{2}+(b^2+c^2-2bc)\sin^2\frac{A}{2}$ 5

$= (b^2+c^2+2bc)\sin^2\frac{A}{2}$ 5

$= (b+c)^2\sin^2\frac{A}{2}$

$\boxed{25}$

(ii) $\alpha+\beta-\gamma=\pi$

$\sin^2\alpha+\sin^2\beta-\sin^2\gamma$

$= 1-\sin^2\gamma+\sin^2\beta-\cos^2\gamma$ 10

$= \cos^2\gamma-\cos(\alpha+\beta)\cos(\alpha-\beta)$ 5

$= \cos^2\gamma+\cos\gamma\cos(\alpha-\beta)$ 5

$= \cos\gamma\left(\cos(\alpha-\beta)+\cos\gamma\right)$

$= \cos\gamma\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)$ 5

$= \cos\gamma\cdot 2\sin\alpha\sin\beta$ 5

$= 2\sin\alpha\sin\beta\cos\gamma$

$\boxed{30}$

(b) (i) $x+y=\frac{\pi}{6}$

$\Rightarrow \tan(x+y)=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ 5

$\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{1}{\sqrt{3}}$ 10

$\Rightarrow \tan x+\tan y=\frac{1-p}{\sqrt{3}}$ 5

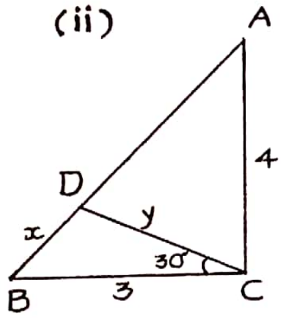
$(\because \tan x\tan y=p)$

$\therefore$ The equation whose roots are $\tan x$ and $\tan y$ is

$x^2-\left(\frac{1-p}{\sqrt{3}}\right)x+p=0$ 10

$\Rightarrow \sqrt{3}x^2-(1-p)x+\sqrt{3}p=0.$

(ii)



AB = 5 unit

$\Rightarrow \sin B=\frac{4}{5}$ &

$\sin A=\frac{3}{5}$ 5

Let $BD=x$, $CD=$

In $\triangle BCD$, sin rule

$\frac{BD}{\sin 30}=\frac{CD}{\sin B}$ 10 $\Rightarrow x=\frac{5y}{8}$ ...(

In $\triangle ACD$, sin rule

$\frac{AD}{\sin 60}=\frac{CD}{\sin A}$ 10 $\Rightarrow 5-x=\frac{5y}{2\sqrt{3}}$ ...

(1), (2) $\Rightarrow \frac{5y}{8}=5-\frac{5y}{2\sqrt{3}}$ 5

$\Rightarrow y=\frac{8\sqrt{3}}{4+\sqrt{3}}$

(c) $\alpha=\tan^{-1}\frac{1}{2x+1}$, $\beta=\tan^{-1}\frac{}{4}$

$\gamma=\tan^{-1}\frac{2}{x^2}$ 5

$\alpha+\beta=\gamma$

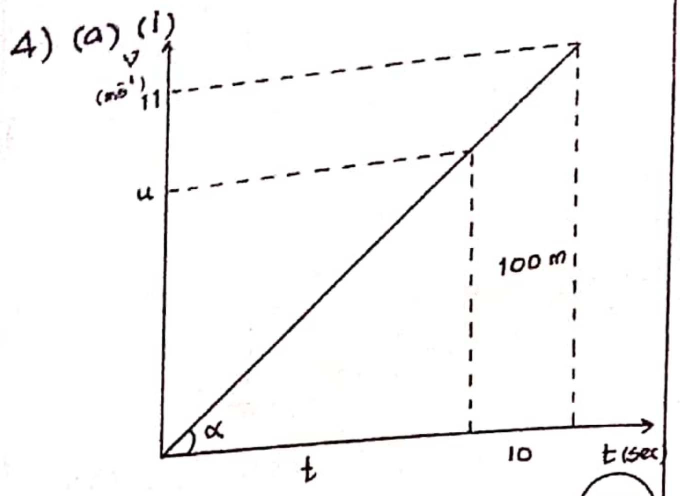
$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\tan\gamma$ 5

$\frac{6x+2}{8x^2+6x}=\frac{2}{x^2}$ 5

$2x(3x+2)(x-3)=0$. 5

$x=0,-\frac{2}{3},3$

4) (a) (i)



(20)

(ii) $\tan\theta = \frac{11-u}{10} = \frac{u}{t} = f$ (10)

$\frac{1}{2}(u+11)\cdot 10 = 100$ (10)

$u = 9\ ms^{-1}$ (5)

$\Rightarrow\ t = \frac{9}{1/5} = 45$ sec (10)

(iii) $f = \frac{1}{5}$ (10)

$S = \frac{1}{2} \times 55 \times 11$ (10)

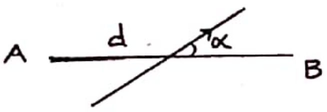
$= 302.5$ m

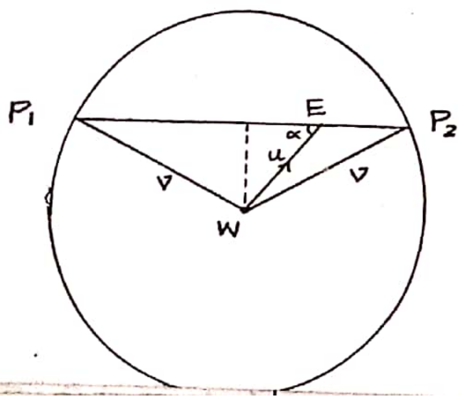
75

(b) $V_{WE} = $ u, α $\quad V_{PE} \rightleftarrows V_{PW}$ ∇

$PE = PW + WE$ (10)

$\rightleftarrows$ ∇ α





(25)

$P_2E = w_2$

$w_1 = \sqrt{v^2 - u^2\sin^2\alpha} + u\cos\alpha$

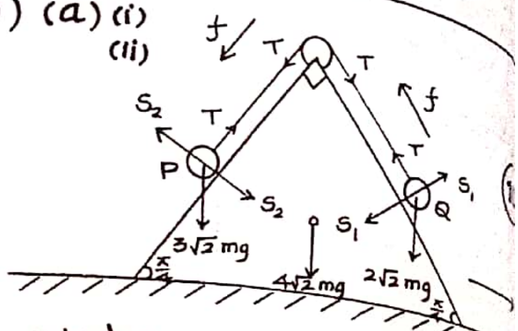
$w_2 = \sqrt{v^2 - u^2\sin^2\alpha} - u\cos\alpha$

$t_1 = \frac{d}{w_1} \qquad t_2 = \frac{d}{w_2}$ (10)

$t_1 + t_2 = d\left(\frac{1}{w_1} + \frac{1}{w_2}\right)$ (5)

$= \frac{2d\sqrt{v^2 - u^2\sin^2\alpha}}{v^2 - u^2}$ (5)

(15) (a) (i)
(ii)



Wedge = W
Particle of mass $3\sqrt{2}m = P$
Particle of mass $2\sqrt{2}m = Q$

$A_{WE} = \rightarrow F \qquad A_{PW} = f\ \frac{\pi}{4} \qquad A_{QW} = f\ \frac{\pi}{4}$

(iii) For the system, $F = ma$

$\rightarrow\ 0 = 4\sqrt{2}\,mF + 3\sqrt{2}\,m(F - f\cos\frac{\pi}{4})$
$+ 2\sqrt{2}\,m\,(F - f\cos\frac{\pi}{4})$

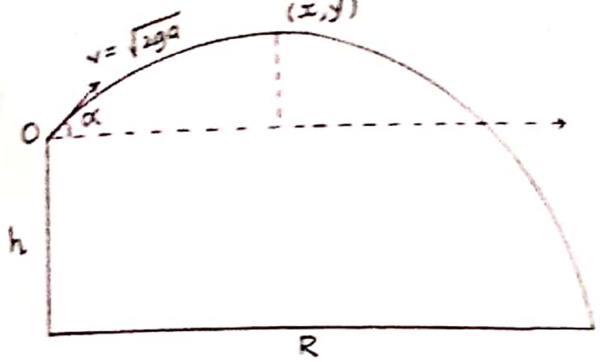
$\Rightarrow\ 9\sqrt{2}F = 5f$

P: $\swarrow\ 3\sqrt{2}\,mg\cos\frac{\pi}{4} - T =$
$3\sqrt{2}\,m\,(f - F\ldots$

Q: $\nwarrow\ T - 2\sqrt{2}\,mg\cos\frac{\pi}{4} =$
$2\sqrt{2}\,m\,(f - F\ldots$

$\Rightarrow\ F = \frac{13}{9}$ (10)

$f = \frac{117\sqrt{2}}{4}$ (5)



$s = ut + \frac{1}{2} f t^2$

$\rightarrow \quad x = v\cos\alpha \, t \quad \ldots (1)$ 5

$\uparrow \quad y = v\sin\alpha \, t - \frac{1}{2} g t^2 \ldots (2)$ 5

$y = \frac{\sin\alpha}{\cos\alpha} x - \frac{1}{2} g \frac{x^2}{v^2\cos^2\alpha}$ 5

$= x\tan\alpha - \frac{\frac{1}{2} g x^2}{2ga} \sec^2\alpha$ 5

$= x\tan\alpha - \frac{x^2}{4a}\sec^2\alpha$

(ii) when $x = R$, $y = -h$ 5

$-h = R\tan\alpha - \frac{R^2(1+\tan^2\alpha)}{4a}$ 5

$R^2\tan^2\alpha - 4aR\tan\alpha + R^2 - 4ah = 0$ 5

For the real values of $\tan\alpha$ 5

$\Delta \geqslant 0$ 5

$16a^2R^2 - 4R^2(R^2 - 4ah) \geqslant 0$ 5

$R^2 \leq 4a(a+h)$ 5

$R_{max} = \sqrt{4a(a+h)}$ 5

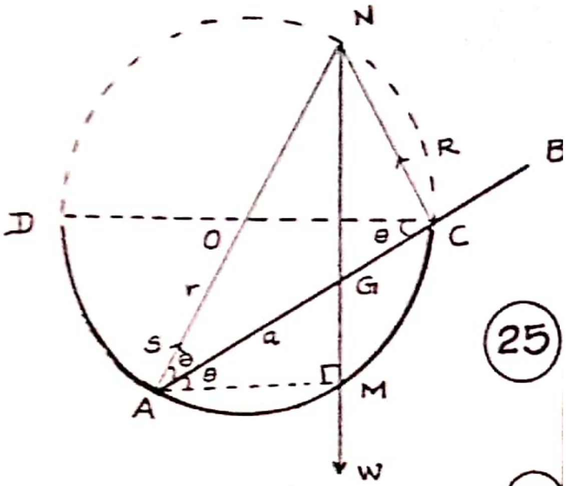
$= 2\sqrt{a(a+h)}$

(iii) $\tan\alpha = \frac{4aR}{2R^2}$ 5

$= \frac{2a}{2\sqrt{a(a+h)}}$ 5

$= \frac{a}{\sqrt{a(a+h)}}$ 5

$\tan^2\alpha = \frac{a}{a+h}$



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AB ↗ $S\cos\theta - W\sin\theta = 0$ 15

$S = W\tan\theta$ 5

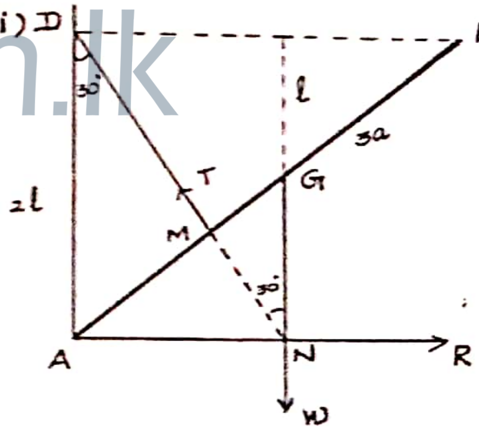
(ii) A ↺ $R \cdot 2r\cos\theta - W \, 2r\cos 2\theta =$

$R = W\frac{\cos 2\theta}{\cos\theta}$ 5

(iii) $AM = a\cos\theta$, $AM = AN\cos$
5
$= 2r\cos 2$

$\therefore \quad a\cos\theta = 2r\cos 2\theta$ 5

(b) (i)



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$\uparrow \quad T\cos 30 - W = 0$ 10

$T = \frac{2W}{\sqrt{3}}$ 5

(ii) $\rightarrow \quad -T\sin 30 + R = 0$ 10

$R = \frac{W}{\sqrt{3}}$ 5

(iii) $\triangle AMD /// \triangle GMN$

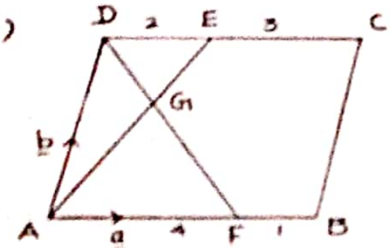
$$\Rightarrow \frac{AD}{GN} = \frac{AM}{MG} = \frac{2}{1} \quad 10$$

$$AM = 2MG$$

$$\therefore \; AM = 2a \quad 10 \quad (AG = 3a)$$

75

7. (a) (i)



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$$\overrightarrow{AG} = \lambda \overrightarrow{AE} \quad 5$$
$$= \lambda (\overrightarrow{AD} + \overrightarrow{DE}) \quad 5$$
$$= \lambda \left(\underline{b} + \tfrac{2}{5}\underline{a} \right) \quad 5$$

(ii) $$\overrightarrow{GF} = \mu \overrightarrow{DF} \quad 5$$
$$= \mu (\overrightarrow{DA} + \overrightarrow{AF}) \quad 5$$
$$= \mu \left(-\underline{b} + \tfrac{4}{5}\underline{a} \right) \quad 5$$

(iii) $$\overrightarrow{AG} + \overrightarrow{GF} = \overrightarrow{AF} \quad 5$$

$$\lambda \left(\underline{b} + \tfrac{2}{5}\underline{a} \right) + \mu \left(\tfrac{4}{5}\underline{a} - \underline{b} \right) = \tfrac{4}{5}\underline{a} \quad 5$$

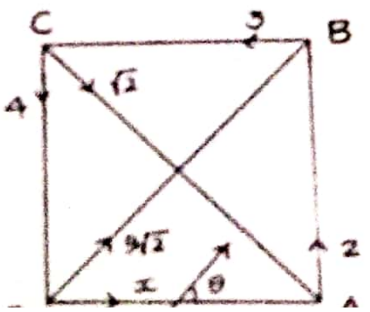
$$\left(\tfrac{2}{5}\lambda + \tfrac{4}{5}\mu - \tfrac{4}{5} \right)\underline{a} + (\lambda - \mu)\underline{b} = \underline{0} \quad 5$$

$$\tfrac{1}{5}(2\lambda + 4\mu - 4) = 0 \quad 5 \quad \& \quad \lambda - \mu = 0 \quad 5$$

$$\lambda = \mu \quad 5 \quad \& \quad \lambda = \tfrac{2}{3} \quad 5$$

70

(b) (i)



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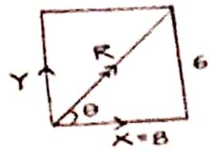
$\overrightarrow{OA}$ $\quad x = 1 - 3 + 9 + 1 \quad 10$

$$= 8 \quad 5$$

$OC \uparrow \quad Y = 2 - 4 + 9 - 1 \quad 10$

$$= 6 \quad 5$$

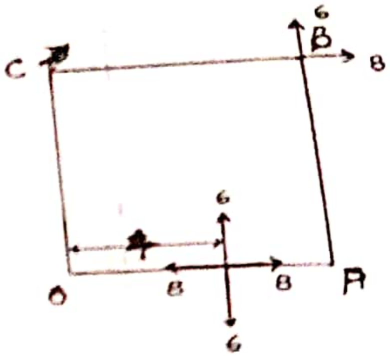
$R = 10\ N \quad 5$



tanθ =

5

(ii)



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$$O \circlearrowleft \quad Y \cdot x = 2 \cdot 6 + 3 \cdot 6 - \sqrt{2} \cdot 6 \sin 45 \quad 10$$

$$6x = 12 + 18 - 6$$

$$x = 4 \quad 10$$

(iii) $\overset{\curvearrowright}{G} = 8 \cdot 6 - 6 \cdot 2$

$$= \overset{\curvearrowright}{36}\ Nm \quad 10$$