G.C.E A/L Support Revision Programme -2013

Revision Paper

Combined Mathematics -1

Ministry of Education

this Paper consists of two parts

Part A consists of 10 questions Answer all questions

Part B consists of 7 questions answer any 5 questions

Time – Three Hours

Part A

1) Sketch the graphs $y = |x + 1|$ and $y = |2x + 1|$ in the same figure. Find the range of values of x that satisfy the inequality $|x + 1| < |2x + 1|$.

2) Consider taking all the letters ECCENTRICTTY. Find the number of permutations that can be formed
   i) With no restrictions what so ever in the order of letters
   ii) With a letter "T" occurring immediate after N.

3) Find the coefficient of x^{2013} in the expansion $(1 + x)^{2012}(1 - x + x^2)$

4) Evaluate i) $\lim_{x\to 0} \frac{\sqrt{1+x-x^2}-1}{x}$ ii) $\lim_{x\to\infty} \frac{1+x-3x^2}{x^2-4x+8}$

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5) Differentiate with respect to x $\quad 3^{(1/x+1)} \quad y = 4$

6) Evaluate $\int_{-2}^{2} |x + 1|\, dx$

7) Find the equation of the bisector of the angle between the two straight lines $3x + 4y + 5 = 0$ and $12x - 5y - 3 = 0$ for the angle which contains the point of origin

8) Find the coordinates of the point P which bisects the two points $A(4\,, 0)$ and $B\ (3, -3)$ externally in the ratio 3:1 .

9) Find the equation of the circle which touches the line $4x - 3y - 6 = 0$ and the co-ordinate axes.

10) Deduce that $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ using the expansions of $\sin(x + y)$ and $\cos(x + y)$.

if $A+B+C=\pi$ show that $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{C}{2} = 1.$

Part B

Answer any 5 questions

11) (a) The graph of $y = x^2 - (k-2)x + k + 1$ intersect the x-axis at two distinct points $(\alpha, 0)$ and $(\beta, 0)$; $k \in \mathbb{R}$

(i) Find the range of values k can take.

(ii) Also if ($|\alpha + \beta| < 5$ Find the range of values k can take.

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(b) Show $\frac{x^3+4x^2-10x+6}{(x-3)(x^2+4)}$ in partial fractions

12) (a)

$u_1, u_2, u_3, \ldots, u_n, \ldots$ is a positive number sequence. They are in an arithmetic series. Prove that $\frac{1}{u_1u_2} + \frac{1}{u_2u_3} + \frac{1}{u_3u_4} + \ldots + \frac{1}{u_{n-1}u_n} = \frac{n-1}{u_1u_n}$ applying the principals of mathematical induction for $n \geq 2$.

(b) Given $v_r = r(r+1)$ and $f(r) = Ar^2 + Br + C$; $r \in \mathbb{Z}^+$. Find the constants A, B and C such that $v_r = f(r) - 3f(r-1)$ hence evaluate $\sum_{r=1}^{n} \frac{v_r}{3^r}$

Is this series convergent? Justify your answer.

13) (a) Given 3 matrices

A=$\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 5 \\ -4 & k \end{pmatrix}$

Find the value of k such that $A(2B - C) = 6I$ where I is the unit matrix.

Hence deduce A^{-1}.

(b) (i) $\left(\frac{3-i}{3+i}\right)Z = \frac{5+5i}{-1+2i}$ express the complex number Z in the form a+ib

(ii) ABCD is a rhombus in the argon diagram with the vertices in the anticlockwise sense. There points represent the complex numbers Z_1, Z_2, Z_3 and Z_4 . ABC $\measuredangle = \frac{\pi}{3}$

Show that $Z_2 = \frac{1}{2}Z_1\left(1 + \sqrt{3i}\right) + \frac{1}{2}Z_3\left(1 - \sqrt{3i}\right)$

And $Z_4 = \frac{1}{2}Z_1\left(1 - \sqrt{3i}\right) + \frac{1}{2}Z_3\left(1 + \sqrt{3i}\right)$.

(iii) R is the region that satisfy $|Z - 4i| \leq 2$ in the argon diagram. Find the difference of the highest volume and the lowest value of $\arg Z$ in this region.

Determine the highest and the lowest values of $|Z|$

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14) (a)

(i) $a(x-y) = \sqrt{1-x^2} + \sqrt{1-y^2}$, $a \in \mathbb{R}$ and " a " is a nonzero constant show that $\tan\frac{\theta-\alpha}{2} = \frac{1}{a}$ by substituting $x = \sin\theta$ *and* $y = \sin\alpha$ such that $\theta + \alpha \neq \pi$.

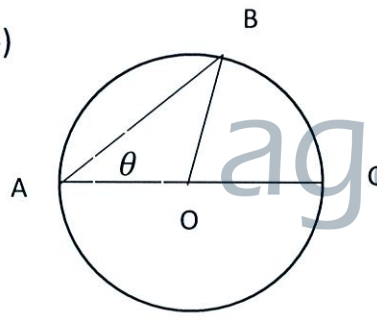
Hence show that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(ii) Is $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ show that

$$(1-x^2)\frac{d^3y}{dx^3} - 5x\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$$

Find $\left(\frac{d^3y}{dx^3}\right)$, $x=0$

(b)



The figure shows a circular lake with radius $2km$ and center O. A is a fixed point AB is a variable chord. The constant speed a person can swim along AB $2\sqrt{3}\ kmh^{-1}$. One can walk along the bank from B to C at a constant speed of of $4kmh^{-1}$. $BAO\measuredangle = \theta$ It takes T(θ) hours for the man to go from A to C by swimming at constant speed from A to B and walking at constant speed from B to C.

Find T(θ). Find the θ,maximum time taken to go from A to C by examining the sign of the first derivative.

15) (i) Find $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3+\cos^2 x}dx$ by a suitable substitution.

(ii) Given $I_n = \int \frac{1}{(a+x^2)^n}dx$ for $n \geq 1$.

Show that $I_{n+1} = \frac{x}{2an(a+x^2)^n} + \left(\frac{2n-1}{2an}\right)I_n$

and hence find $\int \frac{1}{(4+x^2)^3}\,dx$.

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16) Equation of the side OA of a Rhombus OABC 4x-3y=0 The equations of the diagonal OB is x – y = 0. O is the origin. Find the Equation of OC .If B = (5 , 5) find the equations of AB, AC and BC.

A circle with radius 1 unit is drawn to touch the sides OC and OA of the rhombus OABC.

Find the equation of the circle.

Find the Equation of the tangent chord of the tangents drawn to the circle from O.

17)

I. State the cosine rule, using the usual notations, for a triangle ABC.

BC, CA and AB of a circle ABC are $x + y$, x , and $x - y$ respectively.

Show that $\cos A = \frac{x-4y}{2(x-y)}$

ii) Show that $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\ cos^2\ \frac{\alpha-\beta}{2}$

Find the general solution of the equation

$(\cos x + \cos 3x)^2 + (\sin x + \sin 3x)^2 = 1$

iii) Solve $2\tan^{-1}(\sin x) - \tan^{-1}(2\sec x) = 0$.

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G.C.E A/L SUPPORT REVISION PROGRAMME -2013

Revision Paper (10) Combined Mathematics II

Ministry of Education

This paper consists of two parts
Part A - Consists of 10 questions (Answer All questions)
Part B – Consists of 7 questions (Answer any 5 questions) Time- Three Hours

Part A

1. Distance between two places A and B on a straight road is 2a.There is a road block c between A and B at a distance $\frac{a}{3}$ from A. A vehicle passes A at a uniform velocity u and proceeds towards B and due to the road block at C suddenly reduces its velocity by w (0<w<u), retards uniformly and comes to rest at B Draw a rough velocity time diagram and hence find the time spent by the vehicle to go from A to B and the retardation.

2. The position vectors of A and B with reference to the point O are respectively $2\boldsymbol{i}+3\boldsymbol{j}$ and $\boldsymbol{i}+\boldsymbol{j}$; $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors along the OX and OY axes.

   a). Find the angle $A\hat{O}B$.

   b). If the position vector of C is $-2\boldsymbol{i}+j$ show that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$.

3. A smooth sphere of mass 3m moves on a smooth horizontal plane along a straight line at a velocity of 4v. Another smooth sphere B of the same size and of mass 2m moves along the same straight line in opposite direction and after simply colliding the sphere A comes to rest.
   (a) Show that the direction in which B moved is the reverse
   (b) Find the coefficient of restitution between the spheres.

4. The mass of a cyclist and his bicycle is 90kg. He moves down from a top of a hill at an inclination $\sin^{-1}\frac{1}{15}$ at a uniform velocity of $36kmh^{-1}$ without padding. Find the resistance to the motion. If the resistance to the motion is proportional to the velocity of the bicycle. Find his power when he travels at a uniform velocity of $24\ kmh^{-1}$ along a horizontal road.

5. A smooth uniform solid hemisphere of weight $2w$ and radius a is kept on a smooth horizontal plane with its curved surface touching the plane. An insect of weight $\frac{w}{2}$ settles quietly on the plane surface of the hemisphere. Then the plane surface of the hemisphere rests at equilibrium at an inclination $\tan^{-1}\frac{1}{3}$ to the horizontal. Find the distance from the center of the plane surface to the point at which the insect settled.

6. AB is a uniform rod of length $2a$ and weight w. It is at limiting equilibrium with the end A touching a rough horizontal plane with coefficient of friction $\frac{1}{\sqrt{3}}$ and a point C on the rod touching a smooth peg. At this stage the rod is inclined 60^0 to the horizontal. Find the distance BC.

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7. The probability of a man and his wife living more than 30 years after marriage is $\frac{1}{3}$ and $\frac{1}{2}$ respectively. The two events are independent events.
Find the probability of
(i) Both man and his wife living more than 30 years after marriage.
(ii) At least one of them living more than 30 years after marriage.

8. The following probabilities are given. S is the sample space. A and B are Two events such that $p(A) = \frac{1}{2}$, $p\left(\frac{A}{B^1}\right) = \frac{2}{3}$ සහ $p\left(\frac{A}{B}\right) = \frac{3}{7}$.
B^1 is the complement of event B.
(i) Find $P(A \cap B)$ and $P(A \cup B)$
(ii) Are the two events A and B mutually exclusive? Justify your answer.

9. Find the appropriate class intervals of the continues distribution given below calculate the mode and the mean.

Class Interval	Mid Value	Frequency
....	30	15
....	35	23
....	40	17
....	45	15
....	50	10

10. The mean and the standard deviation of a collection of data are 42 and 15 respectively. Data has been re casted so that the mean is 50 and the standard deviation is 20. Find the revised value relevant to the original value 54.

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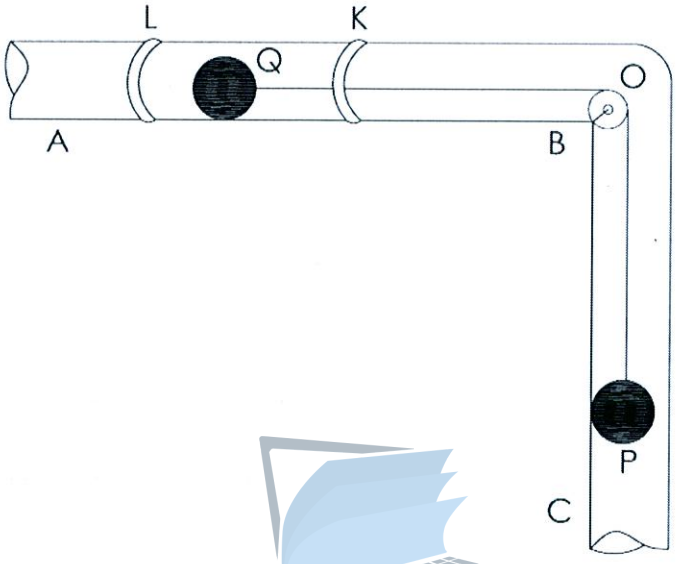
Part B

11. (a) A motor boat with a maximum speed of u kmh^{-1} Needs to catch a ship sailing in the North West direction at a speed of v kmh^{-1} at the beginning the ship was d km south of the motor boat. Find the direction in which the motor boat should move in order to catch the ship. Use a velocity triangle.
Also find the time taken to catch ship after the first observation.
(b)



Shown in the figure is a smooth tube ABC of mass M, Right angled at B The tube passes through two smooth rings K and L. There is a smooth pulley O fixed at B. There are two particles P and Q with mass of each m at the two ends of an inelastic string that passes through the pulley. Find the acceleration of the particles due to the moment, relative to the tube, when the system is released freely.
Find the reaction between the particle p and the tube.

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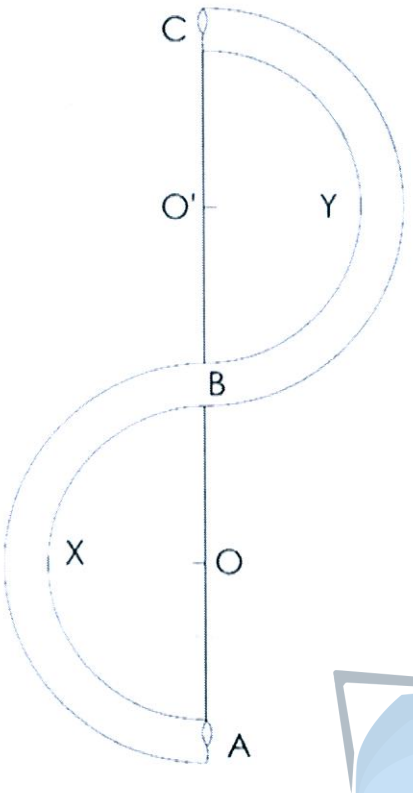
12.



Given in the figure is a combination of two semicircular narrow tubes AXB and BYC both with radius $\boldsymbol{a}$. C lies above A . A particle **P** of mass $\boldsymbol{m}$ is projected horizontally into the tube with a velocity $\boldsymbol{u}$ at A. When $AOP\measuredangle = \theta$ the velocity of the particle is $\boldsymbol{v}$ and the reaction on the particle by the tube is R.

Show that $v^2 = u^2 - 2ga(1 - \cos\theta)$ and $R = \frac{m}{a}(u^2 - 2ga + 3ga\cos\theta)$

Show that if $u^2 < 4ag$ the particle cannot enter the upper semicircular tube.

Find the velocity V' of the particle and the reaction R' of the tube on the particle if $u^2 = 10ag$ when BO'P $\measuredangle = \alpha$

Find at what distance from A, on the horizontal plane through A , does the particle that exit at C drop.

13. One end of an elastic string whose natural length is $\boldsymbol{a}$ and the modules of elasticity is $\boldsymbol{mg}$ is joined to a fixed point C at a distance $\boldsymbol{2a}$ from the edge of a table. A particle p of mass $\boldsymbol{m}$ is fixed to the other end of the string. One end of an inelastic string is tied to particle p and a particle Q of mass $\boldsymbol{m}$ is fixed to the other end. Particle P is kept at a distance $\boldsymbol{a}$ from C at the beginning. The inelastic string hangs from the edge of the table, with particles Q and perpendicular to the edge of the table.

When the system is released from rest, the length of the elastic string is x in time t $(x \leq 2a)$

Using the usual notation show that $\ddot{x} + \frac{g}{a}(x - 2a) = 0$,

Taking $x - 2a = A\cos\omega t + B\sin\omega t$

Find the constants A and B; here $\omega = \sqrt{\frac{g}{a}}$

Find the maximum length of the elastic string can take and the tension on the inelastic string. ($\boldsymbol{g}$ is the acceleration due to gravity.)

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14. (a) A particle of weight w is fixed to a point P on a circular plate with center O ,radius r and weight w'. Two points on the circumference of the plate are at limiting equilibrium touching, and perpendicular to the rough horizontal floor and the rough vertical wall. P lies above O. The horizontal distance from the wall to P is $\frac{r}{2}$ .The angle of friction at both points of contact is λ .

(i) Show that $\cos 2\lambda - \sin 2\lambda = \frac{w}{w+w'}$

(ii) Find λ if $w' = w(\sqrt{2} - 1)$

(b) Forces $\lambda, 2\lambda, \mu, \mu, 2\lambda$ and λ act along $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EF}$, and $\overrightarrow{FA}$ of a regular hexagon ABCDEF with side a.

The resultant force of the system passes through E. Find the value of μ in terms of λ.

Show that the resultant of the system of forces passes through C and is $8\sqrt{3}\lambda$.

Find the distance from A to the point at which the line of action of the resultant meets AB, when a couple of forces act on the same plane in the direction BAF with moments of $4a\lambda\sqrt{3}$ are applied to the system of forces.

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15. (a) Four uniform rods of weight w each are hanged smoothly to form a rhombus, ABCD. The system is suspended freely from D. A and the midpoint of CD are joined by a light rod such that $A\hat{D}C = \frac{\pi}{3}$ .Find the reaction of joint C , and the force of the light rod.

(b) Four equal light rods are hanged roughly to form a rhombus. A and C are joined by a light rod Show that $B\hat{A}D = \frac{\pi}{3}$. The structure is suspended by two inelastic strings tied to points B and D. A weight $2w$ is hung at C Show that A is above C .When the system is in equilibrium in a vertical plane the two strings makes of $\frac{\pi}{6}$ to the horizontal.

Find the tensions of the strings.

Use Bo Notation and draw a stress diagram only for the joint B.

Hence show the stress of AB, BC CD and DA are whether tensions or compressions and find them.

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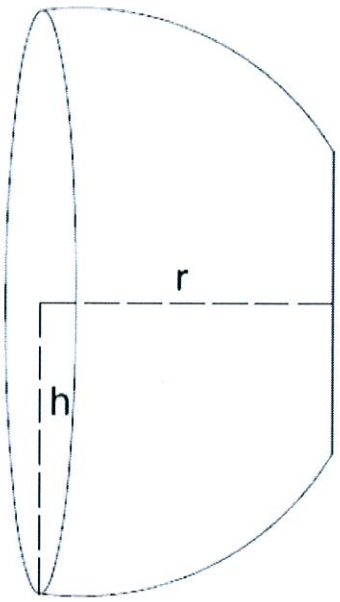
16.



The figure shows a solid taken out from a uniform solid hemisphere of radius r using a plane parallel to the plane surface , perpendicular to the to the axis of symmetry and at a distance h from the center of the plane surface $h(\leq r)$

Find the center of gravity of the solid using integration.

Thus deduce the position of the Centre of gravity of a solid hemisphere.

The solid shown in the figure is freely hung from a point on the circumference of the small circular plane. Find the inclination of the small circular that face to the horizontal.

17. (a) $x_1, x_2, x_3, \ldots\ldots, x_n$ are in observations from a population. Define its sample mean $\bar{x}$ *and* the sample variation s_x^2

$y_1, y_2, y_3, \ldots\ldots, y_m$ are observations from another population. $\bar{y}$ and s_y^2 are it's sample mean and the sample variation. When both populations are taken together and the sample mean is $\bar{z}$ and the sample variation is s_z^2 show that

$$\bar{z} = \frac{n\bar{x}+m\bar{y}}{n+m} \text{ and ,}$$

$$s_z^2 = \frac{ns_x^2+ms_y^2}{n+m} + \frac{n(\bar{z}-\bar{x})^2+m(\bar{z}-\bar{y})^2}{n+m}$$

Shown below is information obtained for two samples A and B selected at random.

	Data	Sample Mean	Sample Variation
Sample A	100	41	9
Sample B	50	38	4

If the two samples are taken together find the mean and the variation of the combined sample.

(b) There are 13 cards in a box. Number 1 is written on 6 cards , number 2 is written on 4 cards and number 3 on the rest. There are containers marked 1, 2 and 3.There are r_1 red marbles and b_1 black marbles in container 1.There are r_2 red marbles and b_2 black marbles in container 2. There are r_3 red marbles and b_3 black marbles in container 3. A card is taken out from the box and then a marble is taken out from the container bearing the number equal to the number on the card taken out from the box earlier. Draw a tree diagram to illustrate the event

(i) Find the probability of the marble taken out being a black marble.

(ii) If the marble taken out is a black marble find the probability of it being one taken out from the container marked number 3.

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